# CombinaTexas 2024
# March 23–24, TAMU

(Central Standard Time Zone)

All the activities will be held at the first floor of Blocker Building on the campus of Texas A&M University. All the plenary talks will be at the main lecture room, BLOC 166. For contributed talks, session A is in BLOC 166, and session B is in BLOC 164. The Registration/Breakroom is at BLOC 140/141.

## Schedule and Program

**Saturday Morning, March 23, 2024**

08:00–08:25: Registration and Breakfast
08:25–08:30: Opening Remarks
08:30–09:20: Plenary Talk 1. Jeremy Martin
09:30–11:05: Contributed Session I
11:05–11:30: Break
11:30–12:20: Plenary Talk 2. Fan Wei

12:30–02:00: Working Lunch

02:00–03:10: Contributed Session II
03:20–04:10: Plenary Talk 3. Jessica Striker
04:20–04:40: Break
04:40–06:15: Contributed Session III

**Sunday, April 24, 2024**

08:00–08:30: Registration and Breakfast
08:30–10:05: Contributed Session IV
10:05–10:30: Break
10:30–11:20: Plenary Talk 4. Boris Bukh
11:30–12:20: Plenary Talk 5. Sam Hopkins

# Schedule for Contributed Talks

| Saturday Morning, Contributed Session I | | |
|---|---|---|
| | Session A | Session B |
| | Chair: Derek | Chair: Chun-Hung |
| 09:30–09:50 | Yichen Ma | Youngho Yoo |
| 09:55–10:15 | Anton Dochtermann | Brittian Qualls |
| 10:20–10:40 | Joshua Swanson | Aiya Kuchukova |
| 10:45–11:05 | Matthew Samuel | Samuel Weiner |
| | | |
| **Saturday Afternoon, Contributed Session II** | | |
| | Session A | Session B |
| | Chair: Catherine | Chair: Jacob |
| 2:00–2:20 | Nathan Williams | Katie Anders |
| 2:25–2:45 | Theo Douvropoulos | Amelia Gibbs |
| 2:50–3:10 | Weston Miller | Dylan Douthitt |
| | | |
| **Saturday Afternoon, Contributed Session III** | | |
| | Session A | Session B |
| | Chair: Jacob | Chair: Youngho |
| 4:40–5:00 | Blake Jackson | Wencai Liu |
| 5:05–5:25 | Jihyun Lee | Boris Apanasov |
| 5:30–5:50 | Amanda Burcroff | Art Duval |
| 5:55–6:15 | Tucker Ervin | Matthew Faust |
| | | |
| **Sunday Morning, Contributed Session IV** | | |
| | Session A | Session B |
| | Chair: Derek | Chair: Youngho |
| 8:30–8:50 | Moxuan Liu | Yiwei Ge |
| 8:55–9:15 | Agastya Goel | Nathan Lindzey |
| 9:20–9:40 | Mat Taylor | William Linz |
| 9:45–10:05 | Alec Mertin | Blake Dvarishkis |

# 1 Plenary Talks

1. Boris Bukh, Carnegie Mellon University

   Title: Almost uniform distribution in the cube and a problem of Erdös

   Abstract: What is the "most uniformly distributed" set in $[0,1]^d$? Historically, this question has been motivated both by applications to numerical algorithms, and by questions of diophantine approximation in number theory. In this talk, I will tell about a new constructions of well-distributed points in the cube, and their link to an old problem of Erdős about points in convex position.

2. Sam F. Hopkins, Howard University

   Title: Upho posets

   Abstract: A partially ordered set is called upper homogeneous, or "upho," if every principal order filter is isomorphic to the whole poset. This class of fractal-like posets was recently introduced by Stanley. Our first observation is that the rank generating function of a (finite type N-graded) upho poset is the reciprocal of its characteristic generating function. This means that each upho lattice has associated to it a finite graded lattice, called its core, that determines its rank generating function. With an eye towards classifying upho lattices, we investigate which finite graded lattices arise as cores, providing both positive and negative results. Our overall goal for this talk is to advertise upho posets, and especially upho lattices, which we believe are a natural and rich class of posets deserving of further attention. Essentially no background knowledge will be assumed, and we also hope to highlight several open problems.

3. Jeremy Martin, University of Kansas

   Title: Chromatic symmetric functions and polynomial invariants of trees

   Abstract: Richard Stanley asked in 1995 whether a tree is determined up to isomorphism by its chromatic symmetric function. This question remains unanswered and frequently keeps the speaker awake at night. Our approach to understanding the strength of the chromatic symmetric function as an invariant is to ask what other information is encoded in it. First, we prove Crew's conjecture that the chromatic symmetric function of a tree determines its generalized degree sequence, which enumerates vertex subsets by cardinality and the numbers of internal and external edges. Second, we prove that the restriction of the generalized degree sequence to subtrees contains exactly the same information as the subtree polynomial, which enumerates subtrees by cardinality and number of leaves. Third, we construct arbitrarily large families of trees sharing the same subtree polynomial, proving and generalizing a conjecture of Eisenstat and Gordon. This is joint work with José Aliste-Prieto, Jennifer Wagner, and José Zamora.

4. Jessica Striker, North Dakota State University

   Title: Promotion and a rotating web of mysteries

   Abstract: Many combinatorial objects with strikingly good enumerative formulae also have remarkable dynamical behavior and underlying algebraic structure. In this talk, we consider the promotion action on rectangular tableaux (of several kinds with shapes of at most 4 rows OR at most 2 columns), and explain its small, predictable order by reinterpreting promotion as a rotation in disguise. We show how the search for this visual explanation of combinatorial dynamics led to the solution of an algebraic problem involving web bases. This framework includes several unexpected combinatorial objects of interest: alternating sign matrices, plane partitions, and the Tamari lattice. This talk is based on joint work with Christian Gaetz, Stephan Pfannerer, Oliver Pechenik, and Joshua Swanson as well as with Rebecca Patrias and Oliver Pechenik.

5. Fan Wei, Duke University

   Title: Graph homomorphism density inequalities

   Abstract: Graph limits is a recently developed powerful theory in studying graphs from a continuous perspective. In this talk, we will show it is undecidable to prove general inequalities of homomorphism densities into (edge weighted) graphs. Then I will show how the perspective of graph limits helps with graph homomorphism inequalities and how to make advances in a common theme in extremal combinatorics: when does randomness give nearly optimal bounds?

## 2 Contributed Talks

(Ordered by the registration time)

1. Youngho Yoo, Texas A&M University. (1)

   Title: Minimum degree conditions for apex outerplanar minors

   Abstract: Motivated by Hadwiger's conjecture, we study graphs $H$ for which every graph with minimum degree at least $|V(H)|-1$ contains $H$ as a minor. We prove that a large class of apex-outerplanar graphs satisfies this property. Our result gives the first examples of such graphs whose vertex cover numbers are significantly larger than a half of its vertices, and recovers all known such graphs that have arbitrarily large maximum degree. Our proof can be adapted to directed graphs to show that every directed graph with minimum out-degree at least $t$ contains a certain orientation of the wheel and of every apex-tree on $t+1$ vertices as a subdivision and a butterfly minor respectively.

2. Nathan Lindzey, Technion – Israel Institute of Technology (5)

   Title: Eigenvalues of Disjointness Graphs

   Abstract: A cornerstone of combinatorics is the Kneser graph defined on the collection of k-sets of an n-element universe such that two k-sets are adjacent if they are disjoint. In 1979, Lovász proved that its eigenvalues are just signed binomial coefficients, leading to an elegant algebraic proof of the celebrated Erdős–Ko–Rado Theorem. Since then, a variety of disjointness graphs for other domains have been studied; however, in many cases, explicit closed-form expressions for their eigenvalues have been elusive. We show how algebraic and topological methods can be used to obtain good combinatorial interpretations and explicit closed-form expressions for the eigenvalues of disjointness graphs, and how these eigenvalue expressions lead to results in extremal and enumerative combinatorics

3. Aiya Kuchukova, Georgia Institute of Technology (9)

   Title: Coloring Locally Sparse Graphs

   Abstract: A graph $G$ is $k$-locally sparse if for each vertex $v \in V(G)$, the subgraph induced by its neighborhood contains at most $k$ edges. Alon, Krivelevich, and Sudakov showed that if a graph $G$ of maximum degree $\Delta$ is $\Delta^2/f$-locally-sparse, then $\chi(G) = O(\Delta/\log f)$. We generalize the notion of locally sparsity by defining graphs $G$ to be $(k, F)$-locally-sparse for some graph $F$ if for each vertex $v$, the subgraph induced by its neighborhood contains at most $k$ copies of $F$. Employing the Rödl nibble method, we prove the following generalization of the above result: for every bipartite graph $F$, if $G$ is $(k, F)$-locally-sparse, then $\chi(G) = O\left(\Delta/\log\left(\Delta k^{-1/|V(F)|}\right)\right)$. This improves upon recent work of Davies, Kang, Pirot, and Sereni who consider the case where $F$ is a path. Our results also hold for list and correspondence colorings in the more general so-called "color-degree" setting.

4. Blake Jackson, University of Connecticut. (10)

   Title: Answering two OPAC problems involving Banff quivers

   Abstract: Cluster algebras are uniquely determined by a skew-symmetric/symmetrizable integral matrix. The importance of skew-symmetric integral matrices (along with an operation called mutation) to the study of cluster algebras has led me and others to study the combinatorics of multidigraphs, mutation, and triangular extensions. In January 2021, Bucher and Machacek posted three conjectures to the Open Problems in Algebraic Combinatorics (OPAC) blog: OPAC-033, OPAC-034, and OPAC-035. These conjectures deal with three infinite classes of quivers (multidigraphs): Banff, Louise, and P quivers. Each of these classes is defined recursively with a large focus on triangular extensions. This presentation will introduce the three families and answer OPAC-034 and OPAC-035 in the affirmative. Specifically, my coauthor and I show that a Banff quiver can be verified to be Banff by only using sinks and sources (OPAC-034) and that every Banff quiver is contained in the class P (OPAC-035).

5. Dylan Douthitt, Louisiana State University. (14)

   Title: A matroid analogue of chordal graphs

   Abstract: A graph is chordal if every cycle of length four or more has a chord. In 1961, Dirac proved that a graph is chordal if and only if it can be built from complete graphs by repeated clique-unions. In this talk, I will describe a generalization of Dirac's result to binary matroids. This talk is based on joint work with James Oxley.

6. Boris Apanasov, University of Oklahoma. (18)

   Title: Bisectors and non-rigidity in Hermitian symmetric rank one spaces

   Abstract: We address G.D.Mostow, L.Bers and S.L.Krushkal questions on uniqueness of conformal or spherical CR structures on the sphere at infinity of non-compact (Hermitian) symmetric rank one spaces compatible with the action of a discrete isometry group. We construct such non-rigid discrete isometry groups whose quotients have infinite volumes and whose non-trivial deformations are induced by equivariant homeomorphisms of the (Hermitian) symmetric space with bounded distortion. This non-rigidity is related to non-ergodic dynamics of our discrete isometry group actions on the limit set which could be the whole sphere at infinity.

7. Nathan Williams, University of Texas at Dallas. (21)

   Title: An elaborate new proof of Cayley's formula

   Abstract: We construct a bijection between certain Deodhar components of a braid variety constructed from an affine Kac-Moody group and vertex-labeled trees on $n$ vertices. By an argument of Galashin, Lam, and Williams using Opdam's trace formula in the affine Hecke algebra and an identity due to Haglund, we obtain an elaborate new proof for the enumeration of the number of vertex-labeled trees on $n$ vertices. This work is joint with MRWAC and UTD students.

8. Samuel Weiner, Louisiana State University. (22)

Title: A Konig Infinity Lemma for Hypergraphs

Abstract: Konig established two versions of his Infinity Lemma. The weaker version says that every connected, locally finite, infinite graph must contain an infinite path as a subgraph. The stronger version says that every such graph must contain an infinite path starting from any specified vertex as an induced subgraph. The purpose of this talk is to obtain the corresponding results for hypergraphs. Moreover, we will detail the minimal infinite path structures for 3-uniform hypergraphs.

9. Moxuan Liu, University of California, San Diego. (24)

   Title: Viennot Shadows and Graded Module Structure in Colored Permutation Groups

   Abstract: An increasing subsequence of a permutation $w \in S_n$ is a sequence of integers $1 \le i_1 < i_2 < \cdots < i_k \le n$ such that $w(i_1) < w(i_2) < ... < w(i_k)$. Longest increasing subsequence in the symmetric group $S_n$ has been a well-studied object in combinatorics. In this talk, we will use orbit harmonics and ideal-theoretical extensions of Viennot's shadow line construction to give an analogous definition of longest increasing subsequence for the colored permutation group $S_{n,r}$.

10. Theo Douvropoulos, Brandeis University. (25)

    Title: Families of Shi-like and Catalan-like deformations of braid and reflection arrangements.

    Abstract: The Shi arrangement $Shi_n$ is a deformation of the braid arrangement $Br_n$ and was introduced by Shi to study Kazhdan-Lusztig cells (that turn out to be unions of the regions of $Shi_n$). It has remarkable numerological and structural properties: it has $(n+1)^{(n-1)}$-many regions that can be naturally labeled by parking functions or trees; it has analogs for all Weyl groups; its characteristic polynomial factors with positive integer roots.

    We will present recent work, joint with Olivier Bernardi, where we give an n-parameter family of deformations of the braid arrangement $Br_n$ that generalize the Shi arrangement $Shi_n$. They share many of the remarkable properties of $Shi_n$, in particular they come with product formulas for their characteristic polynomials, and their regions are naturally labeled by Cayley trees.

    We will finish with generalizations to reflection groups and Catalan deformations, and the original motivation for this construction, coming from the theory of multi-arrangements.

11. Agastya Goel, Euler Circle (26)

    Title: RSK-Complete Cycle Decompositions

    Abstract: We characterize the class of cycle types that can achieve all Young tableau shapes (except the trivial ones with a single row or a single column) under the Robinson–Schensted (RS) correspondence, a property that we call RS-completeness. We prove that for even $n$, cyclic permutations comprise the only fixed cycle type that is RS-complete. For odd $n$, cyclic permutations and almost cyclic permutations which have a cycle of length $n-1$ are the only RS-complete cycle types.

12. Matthew Samuel, PGIM Fixed Income. (28)

    Title: Coproducts of double Schubert polynomials

    Abstract: In a 1998 paper, Bergeron and Sottile proved positivity of the coproduct coefficients of Schubert polynomials by identifying them with certain structure constants of Schubert polynomials. We generalize this result to double Schubert polynomials. In the process, we present our product-coproduct duality theorem, which shows that every structure constant of (double) Schubert polynomials occurs as an easy to identify coproduct coefficient. Linking back to Bergeron and Sottile's work, for ordinary Schubert polynomials this allows reduction of a hierarchy of multiplication rules for Schubert polynomials to multiplying by a very limited class of Schubert polynomials. We are currently at level 0 of the hierarchy, which is the separated descents case announced by Knutson and Zinn-Justin in 2019. When a combinatorial formula for multiplying a Schubert polynomial by a Schur polynomial is found, which may be imminent, we will be at level 1.

13. Mat Taylor, University of Texas at Tyler. (32)

    Title: Counting Pattern-Avoiding Permutations By Cycles

    Abstract: We say a permutation avoids a patter if no set of elements in the permutation, with the general order preserved, contains the same relative ordering as the pattern. This paper investigates and enumerates the number of cycles, $k$, in a given permutation of length $n$ that avoids certain pattern pairs or triples of length three. This talk will cover the cases of avoiding the pattern pair 231, 312 and the pair 123, 132.

14. Blake Dvarishkis, Texas State University. (33)

    Title: Edge-Monic Contributors for the Hypergraph Laplacian and Contributor Duality

    Abstract: Locally signed graphic techniques are used to study all integer matrices via the incidence structure of an oriented hypergraph. Vertex-permutation clones, as determined by their incidence structure, generalize Sachs' Theorem to oriented hypergraphic Laplacians. All coefficients of the total-minor (characteristic) polynomial are characterized through edge-monic classes of these permutation clones. Moreover, the diagonal entries of the Laplacian provide a duality theorem that allows us to reclaim the isospectrality for any Laplacian and its dual.

15. Yichen Ma, Cornell University. (36)

    Title: Hopf monoids and Invariants on Partial Orders and Convex Geometries

    Abstract: We consider a Hopf monoid of partial orders and another of convex geometries, and investigate combinatorial invariants constructed from characters on them. Each invariant comes in a pair consisting of a polynomial and a (more general) quasisymmetric function.

    For partial orders we obtain the order polynomial of Stanley and the enriched order polynomial of Stembridge. For convex geometries we obtain polynomials first introduced by Edelman-Jamison and Billera-Hsiao-Provan. We obtain reciprocity results satisfied by these polynomials from the perspective of characters in a unified manner.

If time permits, we also describe the coefficients of the quasisymmetric invariants as enumerating faces on certain simplicial complexes. These include the barycentric subdivision of the CW-sphere of a convex geometry introduced by Billera, Hsiao and Provan.

16. Amelia Gibbs, Trinit University. (38)

    Title: Two Combinatorial Interpretations of Rascal Numbers

    Abstract: Anggoro et al. defined a Pascal-like triangular sequence, which they call the *Rascal Triangle*. We call the entry in the $n$-th row and $k$-th column of the Rascal Triangle as the $n, k$-th *Rascal number*, denoted $R_{n,k}$. They were originally defined by the recurrence

    $$R_{n,k} = \frac{R_{n-1,k}R_{n-1,k-1} + 1}{R_{n-2,k-1}}$$

    and Fleron later showed them to also satisfy the recurrence

    $$R_{n,k} = R_{n-1,k} + R_{n-1,k-1} - R_{n-2,k-1} + 1.$$

    We provide two combinatorial interpretations of this sequence and use them to prove both recurrences and some identities. We also provide a bijection between our two combinatorial interpretations and offer a natural generalization satisfying a generalized version of the recurrence discovered by Fleron. This generalization was independently discovered by Gregory et al. who provided a combinatorial interpretation which we show is in bijection with ours. We also provide combinatorial proofs for some other sum identities, including one that proves a conjecture of Gregory et al.

17. Weston Miller, University of Texas at Dallas (39)

    Title: Rational Catalan Numbers for Complex Reflection Groups

    Abstract: The spetsial complex reflection groups are complex reflection groups that behave as if they were the Weyl group for some connected reductive algebraic group. Analogs of unipotent characters, Harish-Chandra theory, and Lusztig's Fourier transform can be defined combinatorially for these groups, allowing some techniques from the representation theory of finite groups of Lie type to be extended to spetsial complex reflection groups.

    In a recent paper, Galashin, Lam, Trinh, and Williams introduced a family of rational noncrossing objects for finite Coxeter groups using distinguished subwords. The proof that these objects are counted by rational Coxeter-Catalan numbers used Hecke algebra traces to compute the point count of braid Richardson varieties. Assuming standard conjectures, I prove that this trace technique extends to irreducible spetsial complex reflection groups. That is, I show that the trace of a power of a Coxeter element still produces a rational Catalan number.

18. Wencai Liu, Texas A&M University. (42)

    Title: Algebraic geometry and combinatorics in spectral theory of periodic graph operators

Abstract: In this talk, we will discuss the significant role of the algebraic geometry and combinatorics in the study of periodic graph operators. I will begin by basics of periodic Schrödinger operators on periodic graphs. Then, I will show the application of algebraic geometry and combinatorics in addressing both spectral and inverse spectral problems associated with these operators.

19. Yiwei Ge, Louisiana State University (43)

    Title: Oriented diameter of near planar triangulations

    Abstract: The oriented diameter of an undirected graph $G$ is the smallest diameter over all the strongly connected orientations of $G$. A near planar triangulation is a planar graph such that every face except possibly the outer face is a triangle. In this talk, we will show that the oriented diameter of all $n$-vertex near planar triangulations (except seven small exceptions) is bounded by $\lceil \frac{n}{2} \rceil$, and the bound is tight. This talk is based on the joint work with Xiaonan Liu and Zhiyu Wang.

20. Katie Anders, University of Texas at Tyler (45)

    Title: Non-standard binary representations and the Stern sequence

    Abstract: We show that the number of short binary signed-digit representations of an integer $n$ is equal to the $n$-th term in the Stern sequence. Various proofs are provided, including direct, bijective, and generating function proofs. We also show that this result can be derived from recent work of Monroe on binary signed-digit representations of a fixed length.

21. Matthew Faust, Texas A&M University (47)

    Title: Floquet Isospectrality of the Zero Potential for Discrete Periodic Schrödinger Operators

    Abstract: We study Floquet isospectrality of the zero potential for the discrete periodic Schrödinger operator acting on functions on the n-dimensional square lattice. The classical Ambarzumyan Theorem states for the square lattice, the zero potential has no trivial real Floquet isospectral potentials. In this talk we will provide explicit complex potentials Floquet isospectral to the zero potential using combinatorial methods.

22. Tucker Ervin, University of Alabama (48)

    Title: All Connected Quivers Have An Unrestricted Red Size of $n - 1$ Or $n$

    Abstract: The unrestricted red size of a quiver is the maximal number of red vertices in its framed quiver after any given mutation sequence. In a 2023 paper by E. Bucher and J. Machacek, it was shown that connected, mutation-finite quivers either have an unrestricted red size of $n-1$ or $n$, where $n$ is the number of vertices in the quiver. We prove here that the same holds for the connected, mutation-infinite case using forks. As such, the unrestricted red size for any quiver equals $n - c$, where $c$ is the number of connected components of the quiver that do not admit a reddening sequence.

23. Anton Dochtermann, Texas State University (49)

Title: Chip-firing and parking functions on hypergraphs

Abstract: Associated to a finite graph $G$ is the set of "$G$-parking functions", integer sequences counted by spanning trees that arise in the theory of chip-firing (and which generalize the classical parking functions). They can also be defined in terms of a monomial ideal $M_G$ whose homological properties have interesting combinatorial interpretations. We extend these constructions to the setting of hypergraphs and study combinatorial aspects of parking functions in this context, employing generalized notions of acyclic orientations and spanning trees. Homological properties of the underlying ideals can be understood in terms of generalized permutohedra. This is joint work with Ayah Almousa and Ben White, as well as an REU project with Timothy Blanton, Isabelle Hong, Suho Oh, and Zhan Zhan.

24. Brittian Qualls, Louisiana State University (50)

    Title: Unavoidable immersions of 4-edge-connected graphs

    Abstract: A graph $H$ is called an immersion of a graph $G$ if $H$ can be obtained from a subgraph of $G$ by repeated liftings, which means replacing two adjacent edges $xy$, $xz$ by one edge $yz$. In this talk, we discuss results on unavoidable topological minors and their analogous results for immersions. In particular, we discuss our main result: that every sufficiently large 4-edge-connected graph contains a doubled cycle of length $k$, $C_{2,k}$, as an immersion.

25. Joshua Swanson, University of Southern California (51)

    Title: Tanisaki witness relations

    Abstract: Super coinvariant algebras with $p$ sets of commuting variables and $q$ sets of anticommuting variables have seen a surge of interest in recent years. A conjecture of Zabrocki gives the trigraded Frobenius series for $(p, q) = (2, 1)$. This would extend celebrated connections between diagonal coinvariants, Macdonald polynomials, and the Shuffle Theorem, and would provide a representation-theoretic model for the Delta Theorem. Even the $(p, q) = (1, 1)$ specialization of Zabrocki's conjecture remains open, though recently Rhoades–Wilson determined the Hilbert series. One approach to completing this specialization involves finding relations between certain harmonic differential forms corresponding to generators of Tanisaki ideals. We present two such families of relations, which are sufficient to solve the 1-form case.

26. Jihyun Lee, University of Alabama, (52)

    Title: A Property of G-Matrices in Rank 3 Quivers

    Abstract: Cluster algebras, introduced by Fomin and Zelevinsky, are commutative algebras with recursively constructed generators and relations. Each cluster algebra has distinguished generators called cluster variables.

    g-vectors serve as a comprehensive model for the combinatorial aspects inherent in any cluster algebra. The study of g-vectors is crucial for understanding the properties of cluster variables.

In this talk, we explore fundamental combinatorial objects coming from cluster algebras, such as quivers, quiver mutation, exchange matrices, C-matrices, and G-matrices.

We then introduce an intriguing conjecture applicable to all types of rank 3 quivers :

For any rank 3 quiver Q and mutation sequence w, there exists k (either 1, 2, or 3) such that each column vector in the G-matrix (g-vector), acts as a solution to a quadratic equation whose coefficients are given by the number of arrows between i and j from the quiver $Q^{[k]}$, after mutating at vertex k.

27. Amanda Burcroff, Harvard University. (53)

    Title: Broken Lines and Compatible Pairs for Rank 2 Cluster Algebras

    Abstract:

    There have been several combinatorial constructions of universally positive bases in cluster algebras, and these same combinatorial objects play a crucial role in proofs of the famous positivity conjecture for cluster algebras. In rank 2, two notable examples are the greedy basis constructed by Li-Lee-Zelevinsky using compatible pairs on Dyck paths and the theta basis constructed by Gross-Hacking-Keel-Kontsevich using broken lines on scattering diagrams. While these two bases have been shown to coincide via an algebraic proof, a combinatorial proof has remained elusive. I will talk about partial progress toward this end in joint work with Kyungyong Lee. In the more general setting of rank 2 quantum cluster algebras, we construct a quantum-weighted bijection between certain broken lines and compatible pairs. For the quantum Kronecker cluster algebra, we extend this bijection to all broken lines appearing in the theta basis.

28. Alec Mertin, Clemson University. (54)

    Title: Toggleability Spaces of Fence Posets

    Abstract: We consider general fence posets and completely describe their order ideal (resp. antichain) toggleability space: the space of statistics which are linear combinations of order ideal (antichain) indicator functions and equal to a constant plus a linear combination of toggleability statistics. This allows us to strengthen some homomesies proven by Elizalde et al. for rowmotion on fences and prove a number of new homomesy results.

29. Will Linz, University of South Carolina (55)

    Title: L-systems and the Lovasz number

    Abstract: For positive integers n and k, an L-system is a collection of k-uniform subsets of a set of size n whose pairwise intersection sizes all lie in in the set L. The maximum size of an L-system is equal to the independence number of a certain union of graphs in the Johnson scheme. The Lovasz number is a semidefinite programming approximation of the independence number of a graph. We survey the relationship between the maximum size of an L-system and the Lovasz number. In particular, we give explicit constructions of graphs derived from L-systems which have very large gap between the Lovasz number and the Shannon capacity, greatly improving on all known constructions.

30. Art Duval, University of Texas at El Paso (56)

Title: Powers of a simplex

Abstract: We are looking for a cellular complex that supports the minimal free resolution of powers of ideals. We focus on the extremal monomial ideal $\mathcal{E}_q$, which maximizes Betti numbers of the minimal free resolution of the $r$-th power of monomial ideals with $q$ generators: $\beta_i(I^r) \leq \beta_i(\mathcal{E}_q^r)$ for any monomial ideal $I$ with $q$ generators. The (homogenization of the) Scarf complex of an ideal is contained in every free resolution of the ideal. We provide a partial description of the Scarf complex of $\mathcal{E}_q^r$, and a complete description of it when $r = 3$. Furthermore, when $r = 3$, we prove that the Scarf complex of $\mathcal{E}_q^r$ in fact supports its minimal free resolution. The proof uses discrete Morse matching.