MATH 663, FALL 2017

SUBFACTORS, KNOTS, AND PLANAR ALGEBRAS

Instructor: Michael Brannan, 502B Blocker, mbrannan@math.tamu.edu.

Lectures: TR 11:10am - 12:25pm. Location: TBD

Office Hours: Monday-Friday, by appointment.

Course Webpage: http://www.math.tamu.edu/~mbrannan/math663/

Course Overview: Operator algebras are generalizations of matrix algebras to the infinite dimensional setting. Their theory, however, becomes much more involved and combines linear algebra and analysis. There are two main classes of operator algebras: C*-algebras and von Neumann algebras. C*-algebras have a more topological flavor (and their theory is thus often referred to as non-commutative topology), while the theory of von Neumann algebras is more measure theoretic and probabilistic in nature. Von Neumann algebras themselves are already very intriguing, but their theory becomes even more interesting if one tries to understand subfactors, i.e., the question how one von Neumann algebra can be embedded into another one. This question was considered by Vaughan Jones in the 1980's, and in doing so he found an amazing link to knot theory. In the end this resulted in a new invariant for knots, the Jones polynomial, and earned Jones the Fields Medal. Whereas the first investigations of the subfactor problem were quite analytical, Jones introduced, motivated by the relation with knots, a more combinatorial and diagrammatical description, which goes under the name of planar algebras. This can, on the one side, be seen as a special example of the more general theories of operads, quantum groups, and tensor categories, but has on the other side a very planar (i.e., non-crossing) structure, which makes it resemble the combinatorics of free probability theory.

The goal of this course will be to give an introduction to this broad circle of ideas. The following is a rough plan of the topics to be covered.

Plan of topics to be covered:

- A brief survey of elementary von Neumann algebra theory - basic definitions, representation theory for II_1-factors, GNS construction, standard form.
- Subfactors, the index of a subfactor, Jones' basic construction, the set of admissible values for the index.
- The Temperley-Lieb algebra and knots.
- The standard invariant of a subfactor and planar algebras.
- Connections to quantum groups, tensor categories.
- How to build a subfactor from a planar algebra.
- The classification problem for subfactors of small index.

Target audience and prerequisites: The subject material of this course is quite interdisciplinary: it invoves ideas from algebra (planar algebras, tensor categories, (quantum) group theory), functional analysis (von Neumann algebras), and geometry (knot theory). As such, I expect that this course will attract an interdisciplinary audience, and I will target my lectures to a broad audience. In particular, although a basic familiarity with some operator algebras will be handy (e.g. what is a von Neumann algebra, GNS construction), I will design my lectures so that students can pick up the relevant concepts as needed.

References:

(1) V. Jones, Index for subfactors, Invent. Math., Vol. 72 (1983), No. 1, 1-25.
(2) V. Jones, and V.S. Sundar, Introduction to subfactors, LMS Lecture Notes 234 (1997).
(3) D. Evans and Y. Kawahigashi, Quantum symmetries on operator algebras, Oxford Univ. Press (1998).
(4) V. Jones, Planar Algebras I, arXiv:math/990902.7
(5) V. Jones, D. Shlyakhtenko, and K. Walker, An orthogonal approach to the subfactor of a planar algebra, Pacific Journal of Mathematics, Vol. 246 (2010), No. 1, 187-197
(6) A. Guionnet, V. Jones, D. Shlyakhtenko, Random matrices, free probability, planar algebras and subfactors, Quanta of maths, 201-239, Clay Math. Proc., 11, Amer. Math. Soc., Providence, RI (2010).