

Course: M689, Spring 2016: Singular solutions in elliptic PDEs and their approximation by finite element methods

Instructor: Dr. Alan Demlow

Course description:

Solutions to partial differential equations often possess singularities, that is, points where the solution or one of its derivatives fails to be defined. Researchers in theoretical PDEs have intensively studied the properties of such singular solutions over the past decades, and their efficient computational resolution has been an important theme in the development of numerical methods. In this course we will study singularities arising in solutions to elliptic boundary value problems posed on domains having edges and corners. The course will have three main parts. In the first we study of singularities from a theoretical perspective. We will learn the structure of singular solutions arising elliptic problems posed on polygonal and polyhedral domains in two and three space dimensions and discuss various ways of measuring their smoothness or regularity. In the second part of the course we use a priori error estimates as a tool to describe the ability of finite element methods to resolve singular solutions. Our treatment will focus on effects of mesh grading on error behavior and error estimates in non-energy norms such as the maximum norm. In the final portion of the course we will study a posteriori error estimation and adaptivity, which are important practical tools for automatically resolving singularities and other multiscale phenomenon in PDEs solutions. We will present the theory of convergence and optimality theory for adaptive FEM and discuss the effects of various singularities on adaptive convergence rates.

The course will focus on gaining an in-depth understanding of the above topics in the context of scalar elliptic model problems. Depending on time constraints and the interests of the audience, we may also briefly consider additional topics such as singularity structure and regularity in Maxwell's and Stokes' equations; anisotropic finite element spaces; singularities in elliptic eigenvalue problems; and singularities arising from problem data such as diffusion coefficients and forcing functions.

Prerequisites: We will review basic definitions related to the finite element method, so a prior graduate-level course in finite element methods is not strictly required. Prior exposure to FEM at the level of MATH 610 would however be helpful. The course will serve as a useful complement to MATH 610 and 661, and in part also to a course in modern PDEs theory such as MATH 612.