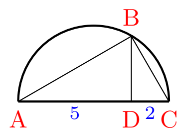
BC Exam
Texas A&M High School Math Contest
October 24, 2015

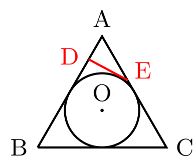
All answers must be simplified, and If units are involved, be sure to include them.

1. $p(x) = x^3 + ax^2 + bx + c$ has three roots, λ_i, with $\lambda_1 + \lambda_2 + \lambda_3 = 5$ and $\lambda_1\lambda_2\lambda_3 = -2$. Suppose that $p(1) = 3$. What is $p(2)$?

2. Triangle ABC is inscribed in a semicircle with side AC being a diameter of the circle. Line BD is perpendicular to AC with AD=5 and DC = 2. What is the length of BD? See the sketch below.



3. A circle centered at O is inscribed in the equilateral triangle ABC. Line segment DE is tangent to the circle, is perpendicular to AB, and intersects AB and AC at points D and E respectively. (See the sketch below) If AD = 1 inch, what is the length of one of the triangle's sides?



4. What does $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{2015^2}\right)$ equal?

5. $p(x) = x^3 + ax^2 + bx + c$ is odd about the point $x = 2$, and $p(1) = 1$. What does c equal?

6. How many different values of the natural number n are there for which $n^2 - 440$ is a perfect square?

7. A park is in the shape of a regular hexagon and a side length of 2 kilometers. Starting at a corner a woman walks along the perimeter a distance of 5 km. How far is she from her starting point?

8. The numbers $a_1, a_2, \cdots a_{2015}$ are consecutive terms of an arithmetic sequence, and

$$\sum_{i=1}^{2015} a_i = a_1 + a_2 + \cdots + a_{2015} = 2015\,.$$

Which, if any, of the a_i can be determined, and what are their values?

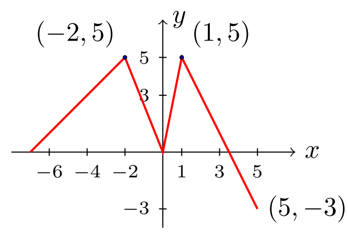
9. A finite sequence of three digit numbers has the property that the tens and units digits of each term are respectively the hundreds and tens digits of the next term. Moreover, the tens and units digits of the last term are respectively the hundreds and tens digits of the first term. One such sequence is

$$245 \quad 451 \quad 512 \quad 124\,.$$

Let S be the sum of the numbers in the sequence. (S for the above sequence is 1332.) What is the largest prime number that divides S, for all possible S?

10. How many different 15 digit numbers can be made up from the integers 1, 2, and 3, assuming that 1 is not in the first five digits, 2 is not in the second five digits, and 3 is not in the last 5 digits?

11. The graph of the function $f(x)$ is shown below. How many solutions does the equation $f(f(x)) = 5$ have?

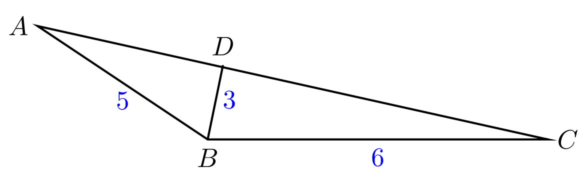


12. Triangle ABC has side lengths $\overline{AB} = 5$, $\overline{BC} = 6$, and $\overline{AC} = 7$. Two bugs starting at the same time from A crawl in opposite directions along the sides of the triangle at the same speed. They meet at point D. What is $\overline{BD}$?

13. The integers 1 through 9 are written on separate pieces of paper, which are then placed in a hat. A slip is drawn, the number on it noted, and then the slip is returned to the hat. This is done one more time. Which integer is most likely to be the units digit of the sum of the two numbers recorded?

14. Simplify the expression
$$\frac{bx\left(a^2x^2 + 2a^2y^2 + b^2y^2\right) + ay\left(a^2x^2 + 2b^2x^2 + b^2y^2\right)}{bx + ay}.$$

15. In the triangle below $\overline{AB} = 5, \overline{BC} = 6$, and $\overline{BD} = 3$. Moreover line $\overline{BD}$ is perpendicular to line $\overline{AC}$. What is the area of triangle ABC?



16. Find the sum of all positive integers n for which $n - 2$ divides $n^2 + 1$. Note, negative integers are allowable divisors.

17. Numbers can be represented in different bases. For example 247 is assumed to be written in base 10 and means $2 \cdot 10^2 + 4 \cdot 10^1 + 7 \cdot 10^0$, while $(247)_8$ (base 8) means $2 \cdot 8^2 + 4 \cdot 8^1 + 7 \cdot 8^0$ or, as normally expressed in base 10, 167. Find the base b such that
$$(43)_{10} = (111)_b\,.$$

18. In the following Sudoku puzzle you are to fill in the missing digits so that each row, column, and small 3×3 squares each contain all of the digit 1 through 9 inclusive. Let a_{ij} denote the digit in the i^{th} row and j^{th} column of the Sudoku puzzle, where a_{11} denoted the entry in the top left hand position, and $a_{1,9}$ denoted the entry in the top right hand position. What is the value of the following sum: $a_{51} + a_{59}$?

9	2					1		
			8		4		2	
4	7	5		9	2		3	
	5	7	4	6	9	8		
	9	4				5	6	
		6	5	2	1	9	7	
	4		6	7		3	9	1
	6		9		3			
		9					8	6