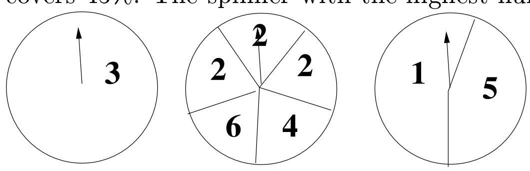
EF Exam

Texas A&M Math Contest

24 October, 2015

(NOTE: If units are appropriate, please include them in your answer. All answers must be simplied where possible.)

1. A game consists of spinning 3 circular spinners (See figure below). Spinner A only has the number 3. Spinner B is divided into 5 equal parts: three of them have the number 2, the other two have 4 and 6 respectively. Spinner C is divided so that the number 1 covers 55% and the number 5 covers 45%. The spinner with the highest number wins. Which spinner is most likely to win?



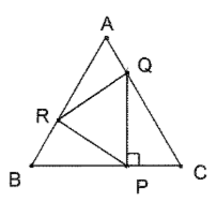
2. A Corps of Cadet freshman started walking from the Quad to the Brazos River at 12noon. Another freshman started walking from the Brazos River to the Quad at 2pm. They passed each other at 4:05pm. Assuming constant speeds throughout, if each reached their destination at the same time, what time was it?

3. Find all values of x such that $(x^3 - 64x)(x^2 + 8x)(9x - 72) \geq 0$.

4. Find the exact value of $\dfrac{76.56 \times \dfrac{8491}{100} - \dfrac{7656}{100} \times 35.72}{49.19 \times \dfrac{634}{1000} + \dfrac{566}{1000} \times 49.19}$.

5. A rhombus is formed by two radii and two chords of a circle of radius 10cm. What is the area of the rhombus?

6. Triangle ABC is equilateral with equilateral triangle PQR inscribed in it. If $\overline{PQ} \perp \overline{BC}$, what is the ratio (in lowest terms) of the area of $\triangle PQR$ to the area of $\triangle ABC$?



7. Find all values of m such that the difference between the roots of $3x^2 + mx - 12$ is 5. What is the product of all such values?

8. Given the system of equations

$$2x - y - 3z = 8$$
$$x - 2y = 7$$

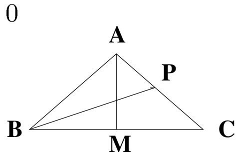
What is the value of $x - y - z$?

9. Evaluate $\dfrac{1 - \tan\left(\frac{\pi}{12}\right)}{1 + \tan\left(\frac{\pi}{12}\right)}$. Simplify your answer completely.

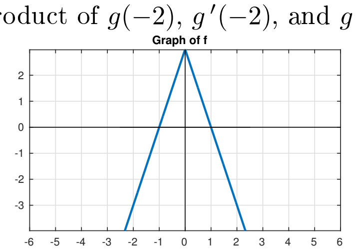
10. If $a > b$ and $x = \dfrac{2(\ln a - \ln b)}{a - b}$, find the numerical value of $a^2 e^{-ax} - b^2 e^{-bx}$.

11. A particle moves along the curve $y = x^2 + 2x$. Find the Cartesian coordinates of the point on the curve where the rate of change for the x and y coordinates of the particle is the same.

12. Given isosceles triangle ABC below with $\angle B = \angle C$ and $BC = 1$. Let point P be the intersection of the line bisecting $\angle B$ and $\overline{AC}$. Determine the length of $\overline{BP}$ as the length of $\overline{AM}$ approaches 0



13. Given the graph of f below consisting of two lines. If $g(x) = \displaystyle\int_0^x (f(t) - 1)\, dt$, what is the product of $g(-2)$, $g'(-2)$, and $g''(-2)$?



14. Find the value of a for which $\displaystyle\lim_{x\to\infty} \left(\frac{x+a}{x-a}\right)^x = e$.

15. Evaluate $\displaystyle\int_0^{\pi^2/4} \cos(\sqrt{x})\, dx$.

16. Rectangle $ABCD$ has vertices at the points $A(0, 0)$, $B(0, 3)$, $C(2, 3)$, and $D(2, 0)$. The rectangle "rolls" along the x-axis, rotating 4 times along its bottom, right corner until $\overline{AD}$ lies back on the x-axis. Find the area of the region which lies above the x-axis and below the curve traced out by the point which starts at $(2, 1)$.

17. The 2015th derivative of $f(x) = \dfrac{1}{x^{2015} - 1}$ can be written in the form $\dfrac{P(x)}{(x^{2015} - 1)^{2016}}$. What is $P(1)$?

18. Given that, for all real values of x, $f(x) > 0$ and

$$(f(x))^2 = \int_0^x \left[(f(t))^2 + (f'(t))^2 \right] dt + 2015$$

write an explicit formula for $f(x)$.

19. Evaluate $\displaystyle\int_1^5 \frac{(x+6)\sin(12-x)}{(x+6)\sin(12-x) + (12-x)\sin(x+6)}\,dx.$