2016 AB Exam

Texas A&M High School Math Contest
October 22, 2016

1. If (x, y) is a point on the circle $x^2 + y^2 = 1$ and the distance from (x, y) to $(0, 1)$ is $\frac{6}{5}$, what is the value of y?

2. A line with slope 2 intersects a line with slope 6 at the point $(40, 30)$. What is the distance between the y-intercepts of these lines?

3. What is the remainder when the polynomial $p(x) = (x - 2)^{2015}$ is divided by the polynomial $q(x) = x - 1$?

4. For each positive integer n let $s(n)$ be the sum of the digits of n. For example $s(23) = 5$, $s(1014) = 6$, etc. What is the minimum value of $s(44 \cdot n)$ where $n = 1, 2, 3, ...$?

5. Find the number of distinct real numbers x which have the property that the median of the five different numbers $x, 6, 4, 1, 9$ is equal to their mean.

6. Which integer is nearest in value to the sum

$$\frac{2007}{2999} + \frac{8001}{5998} + \frac{2001}{3999}?$$

7. Find all numbers m such that the lines $y = x - 2$ and $y = mx + 3$ intersect at a point whose coordinates are both positive.

8. A point (x, y) is called an integer point if both x and y are integers. How many points in the graph of

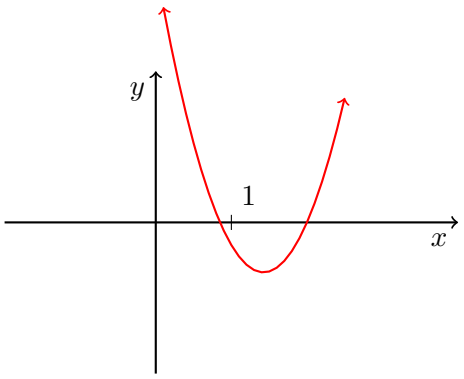
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$$

are integer points? For example, $(2, -4)$ is one such point.

9. Consider the equation $x^2 + px + q = 0$. If the roots of this equation differ by 2, find q in terms of p.

10. The graph of $y = ax^2 + bx + c$ is as shown below. How many of the following expressions are positive?

$$ab, \quad ac, \quad b, \quad a + b + c, \quad a - b + c$$



11. One and only one of the following integers does not divide

$$2^{1650} - 1.$$

Which integer is it?
(a) 3 (b) 7 (c) 31 (d) 127 (e) 2047

12. Find the value of the product

$$(1 - \frac{1}{2^2})(1 - \frac{1}{3^2})(1 - \frac{1}{4^2})...(1 - \frac{1}{99^2})(1 - \frac{1}{100^2}).$$

13. The curve $y = x^4 - 8x^3 + 9x^2 + 20x + 2$ intersects the line $y = 2x + 1$ at four distinct points. Find the average y-value of the intersection points.

14. The sum of the squares of three prime numbers is 182. What is the sum of the three primes?

15. Duke had an average score of 85 on his first eight quizzes. He had an average score of 81 on his first nine quizzes. What score did he receive on his ninth quiz?

16. We have a collection of balls, each labelled $1, 2, 3, ...,$ or 100. These balls are placed in a jar. After repeated trials it is determined that the probability of selecting a ball from the jar labelled n equals n times the probability of selecting a ball labelled 1. What is the probability of slecting a ball labelled 50?

17. Find the smallest integer $n \geq 100$ such that $n^2 + 4n + 2$ is divisible by 7.

18. How many 3-element subsets of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are there such that they contain at least two consecutive integers?

19. Find the smallest positive integer n such that when n is divided by 5 there is a remainder of 1 and when n is divided by 6 there is a remainder of 2.

20. Find the largest integer k such that 135^k divides 2016!
($n!$, n factorial, is defined as $n! = 1 \cdot 2 \cdots n$).