# Best Student Exam

**Texas A&M Math Contest**

22 October, 2016

(NOTE: If units are appropriate, please include them in your answer. All answers must be simplied where possible.)

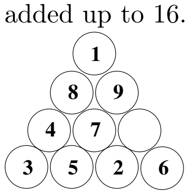
1. Let $A$ be the sum of the distinct prime factors of 20! and $B$ be the sum of the distinct prime factors of 16!. What is $A + B$?

2. Normally, you would leave work at 5:00pm, but being the math nerd you are, you want to wait until the hour and minute hand of the analog clock are at right angles to each other. To the nearest minute, what time do you leave?

3. A block of cheese is 16 inches long, 8 inches wide, and $7\frac{1}{2}$ inches deep. What is the largest number of whole pieces measuring $5 \times 3 \times 2\frac{1}{2}$ inches which can be cut from it?

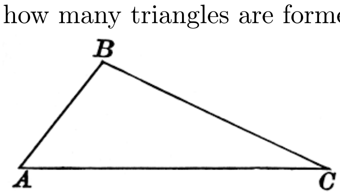
4. Three pirates find a chest of gold coins which are to be divided between them. Before dividing, one pirate secretly counts the number of coins and finds that there is one left over after dividing into three equal piles. Stereotypically, the pirate adds the coin to one pile and steals that pile. Later, the second pirate secretly counts the remaining coins, again finds one left over, and does the same thing. Likewise the third pirate. Later, when the pirates meet, they discover that the remaining coins are evenly divisible into three piles. What is the fewest number of coins that were originally in the chest?

5. A merchant had 10 barrels of sugar. One barrel was not marked; the others had numbers 1-9. The merchant placed the barrels in a pyramid as shown below and noticed that the numbers along each side added up to 16.

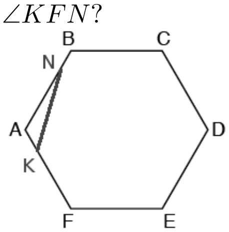



If the barrels are rearranged with the sum of all sides still equal, what is the smallest such sum possible?

6. Given triangle $ABC$ below. If line segments are drawn from $B$ to 2016 different interior points of $\overline{AC}$, how many triangles are formed?




7. On regular hexagon $ABCDEF$, points $K$ and $N$ are chosen on $\overline{AF}$ and $\overline{AB}$ respectively such that $AK + AN = AB$. What is the value (in degrees) of $\angle KAN + \angle KBN + \angle KCN + \angle KDN + \angle KEN + \angle KFN$?



8. It can be easily shown that $2016 = 44^2 + 8^2 + 4^2$. Given this, it can also be shown that $2016^2 = a^2 + b^2 + c^2$, where $a$, $b$, and $c$ are positive integers. Assuming without loss of generality that $a > b > c$, what is the value of $c$?

9. Given five weights, each a different number of ounces, it is possible to combine one or more of these weights on a balance scale to measure out various amounts of salt. For example, with 1, 2, 3, 4, and 5-ounce weights, any integer amount of salt from 1 ounce (using only the one-ounce weight) to 15 ounces (using all 5 weights) can be measured. A merchant claims with their five weights they can measure out any integer amount up to 121 ounces. What is the heaviest weight they have?

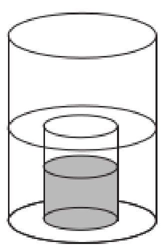
10. Given $\tan\alpha + \tan\beta = 20$ and $\cot\alpha + \cot\beta = 16$, what is $\tan(\alpha + \beta)$?

11. A farmer goes to market and buys one hundred animals at a total cost of \$10,000. Cows cost \$500, sheep cost \$100, and rabbits cost \$5. Assuming the farmer buys at least one of each type of animal, how many cows do they buy?

12. Find the smallest value of $x$ for which

$$\log_{5x+9}\left(x^2 + 6x + 9\right) + \log_{x+3}\left(5x^2 + 24x + 27\right) = 4$$

13. A large cylindrical can with diameter 20cm is partially filled with water. A smaller cylindrical can with diameter 10cm and height 16cm is pushed vertically into the water, which eventually flows over its top. When the smaller can sits on the bottom, it is half full of water (see figure below). What was the original depth of water in the large can?



14. Given four different nonzero digits, form as many different four-digit numbers as possible. Let $S$ be the sum of all these four-digit numbers. What is the largest prime number guaranteed to be a factor of $S$?

15. Given $f(x) = \dfrac{2x^3}{x^2 - 1}$, find $f^{(2016)}(2)$ (i.e., find the 2016th derivative of $f$ at $x = 2$)

16. Let $a_2$, $a_3$, $a_4$, $\cdots$ be an infinite sequence (list of numbers) such that $a_n = 2\ln(n) - \ln(n^2 - 1)$. It can be shown (though not required here) that the sum of all these numbers $\sum_{n=2}^{\infty} a_n$ is a finite number. Find this number.

17. Points $A$, $B$, and $C$ are chosen on the graph of $x^3 + 3xy + y^3 - 1 = 0$ which form an equilateral triangle. Find the area of the triangle.