DE EXAM
## Texas A&M High School Math Contest
October 2017

Directions: If units are involved, include them in your answer.

1. Simplify $\left(a - \dfrac{1}{a}\right)\left(a^2 + 1 + \dfrac{1}{a^2}\right)$.

2. Find the value $\log_b a^2 \cdot \log_a b^2$.

3. If the radius of a circle is increased by 100 % the area is increased by a %. Find a.

4. Find the value of c so that the vertex of a parabola $y = x^2 - 8x + c$ is a point on the x-axis.

5. A regular hexagon is inscribed in a circle. Find the ratio of the length of a side of the hexagon to the length of the corresponding arc.

6. For every natural number n, the sum S_n of n terms of an arithmetic progressions $3n + 4n^2$. Find the kth term.

7. As shown in the figure below, $\angle BAC = \angle BCD$, $AC = 4$ inches, $BC = 5$ inches, $AB = 6$ inches, and $CD = 7.5$ inches. Find BD.
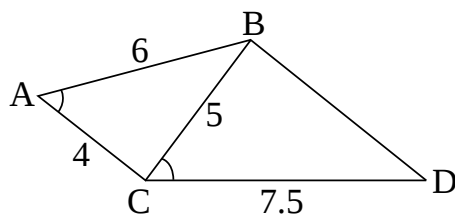
8. Suppose two poles $20''$ and $80''$ high are $100''$ apart. Find the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole.

9. Find x if $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}$.

10. Find the intersection point(s) of graphs $y = \log 2x$ and $y = 2 \log x$.

11. If r and s are the roots of the equation $ax^2 + bx + c = 0$, express $\dfrac{1}{r^2} + \dfrac{1}{s^2}$ in a, b and c.

12. Find the last digit of the number

$$A = 142^1 + 142^2 + 142^3 + \cdots + 142^{20}.$$

13. How many integers satisfy the inequality $x^2 - x - 1980 < 0$?

14. Find $\left(r^3 + \dfrac{1}{r^3}\right)^{2017}$ if $\left(r + \dfrac{1}{r}\right)^2 = 3$.

15. It takes 30 minutes for a man to commute by his car. If he speeds up by 20% how long will it take?

16. Simplify
$$\frac{1^2}{1^4+1} + \frac{2^2-1}{2^4+2} + \frac{3^2-2}{3^4+3} + \cdots + \frac{1000^2-999}{1000^4+1000}.$$

17. Given points $A = (0,1)$, $B = (3,2)$ and an arbitrary point P on the x-axis. Find the minimal value for the length $AP + PB$.

18. Find the maximum of $-x - \dfrac{1}{x}$ for $x > 0$.

19. Simplify $\cos 10° + \cos 20° + \cdots + \cos 180°$.

20. Suppose the hour hand and minute hand of a clock make an angle of $135°$. Assuming the hours and minutes are integers, what is the time? Write your answer in the form $h : m$, where h and m denote hours and minutes, respectively.

21. Suppose a cube has side 1 foot and moves 10 ft along a straight path. Assume that it rains and all rain drops of the same size fall vertically with the same speed of 1 ft/s. Assume further that they are small enough so that the number of rain drops is the same in each cubic feet. The amount of rain that the cube gets (on the front and top faces) when it travels with speeds of 2 ft/s is $a\%$ of the amount when traveling 1 ft/s. Find a.

22. Solve the equation $x + \sqrt{25 - 6x} - 3 = 0$.