Directions: Answers should be simplified, and if units are involved include them in your answer.

1. Solve the equation $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$.

2. A set containing precisely two elements is called a *doubleton*. In how many different ways can we choose three doubeton subsets of $\{1, 2, 3, 4, 5, 6\}$ in such a way that every pair of them has exactly one common element?

3. Chris thinks of three different prime numbers, and notices that their product is equal to 19 times their sum. Compute the sum of the three prime numbers Chris must be thinking of.

4. Let
$$x = \frac{18}{(1 + \sqrt{19})(1 + \sqrt[4]{19})(1 + \sqrt[8]{19})(1 + \sqrt[16]{19})(1 + \sqrt[32]{19})(1 + \sqrt[64]{19})}.$$
Find $(1 + x)^{128}$.

5. In equilateral pentagon $HOWDY$, $\angle H = 90°$ and $\angle W = 120°$. Compute $\angle Y$ (in degrees).

6. Evaluate the following limit.
$$\lim_{x \to 1} \left(\frac{3}{1 - \sqrt{x}} - \frac{2}{1 - \sqrt[3]{x}} \right)$$

7. Let $N = 10004000600040001$. Find $\lfloor \sqrt[4]{N} \rfloor$, where $\lfloor x \rfloor$ denotes the largest integer that is not greater than x.

8. Simplify the expression $S = \sqrt[3]{9 + 4\sqrt{5}} + \sqrt[3]{9 - 4\sqrt{5}}$.

9. Let
$$P_n = \frac{7}{9} \times \frac{18}{20} \times \frac{33}{35} \times \cdots \times \frac{2n^2 + n - 3}{2n^2 + n - 1},$$
for $n \geq 2$. Find $\lim_{n \to \infty} P_n$.

10. What are the last two digits of the number 2019^{2019}?

11. We pick a random point (x, y, z) with integer coordinates from the cube $S = \{(x, y, z) : |x|, |y|, |z| \leq 10\}$. What is the probability of the event that
$$1/x + 1/y + 1/z = 1/(x + y + z)?$$

12. Let $P(x) = (x - 1)(x - 2)(x - 3)$. For how many polynomials $Q(x)$ does there exist a polynomial $R(x)$ of degree 3 such that $P(Q(x)) = P(x) \cdot R(x)$?

13. Suppose that each of 2019 people knows exactly one piece of information, and all 2019 pieces are different. Every time person "A" phones person "B", "A" tells "B" everything he knows, while "B tells "A" nothing. What is the minimum number of phone calls between pairs of people needed for everyone to know everything?

14. Point V lies in the interior of rectangle $JAWS$. The feet of the altitudes from V onto sides $\overline{SJ}$, $\overline{WS}$, $\overline{AW}$, and $\overline{JA}$ are T, I, R, E respectively. Given that $[EAST] = 127$, $[TEAR] = 160$, and $[STAR] = 187$, compute $[JARVIS]$. *(The square brackets denote area.)*

15. Find the sum of the series

$$\sum_{n=1}^{\infty} \arctan\left(\frac{1}{2n^2}\right).$$

16. Find the smallest natural number n such that $2^n + 3^n$ is divisible by 125.

17. Compute

$$\int_0^{\sqrt{\pi}} (4x^4 + 3)\sin(x^2)\, dx.$$

18. Find the multiplicity of 2 in $A = \lfloor (1 + \sqrt{3})^{2019} \rfloor$, i.e., find a whole number k such that A is divisible by 2^k, but not by 2^{k+1}. Here, $\lfloor x \rfloor$ denotes the largest integer that is not greater than x.

19. Find $\displaystyle\lim_{n\to\infty} \frac{n}{(1 \times 2^2 \times 3^3 \times \cdots \times n^n)^{2/n^2}}$.