

CD Exam

TAMU High School Contest

11/04/2023

1. In $CD + EXAM = 2023$, all letters correspond to different digits, $C \neq 0$, $E \neq 0$. Among all solutions, find the maximal possible value of $EXAM$.
2. Each year, the sales went up by $x\%$ in comparison with the previous year. In 2020, the sales were more than \$100000, and in 2022, the sales were 44% more than in 2020. Find x .
3. Out of students who attend a chess club, more than 40% but less than 50% also play tennis. What is the minimal possible number of students in the chess club?
4. S_1 is the sum of even numbers between 1 and 99, and S_2 is the sum of odd numbers between 100 and 200. Find $S_2 - S_1$.
5. Compute:

$$\left(1 - \frac{1}{101^2}\right) \left(1 - \frac{1}{102^2}\right) \cdots \left(1 - \frac{1}{200^2}\right).$$

6. Alice has six sticks with lengths 4, 5, 6, 12, 13, 14. She wants to choose three sticks and make a triangle out of them. How many different triangles she can get?
7. Let ABC be an equilateral triangle with side 1. Let $ABB'A''$, $BCC'B''$, and $CAA'C''$ be squares constructed on the sides of ABC outside of ABC . Find the area of the hexagon $A'A''B'B''C'C''$.
8. How many three-digit numbers satisfy the following property: two of their digits are equal, and the third one differs from these by 1?
9. Find the sum of all digits of the number $4^7 \cdot 5^{10}$.
10. The average age of three siblings Alice, Bob, and Charlie is x . It will become $2x$ when Charlie gets as old as Bob is now. It will become $1.5x$ when Bob gets as old as Alice is now. It will become 30 when Charlie gets as old as Alice is now. Find Bob's age.
11. A configuration of planes and points contains 12 planes and 32 points. Each plane contains 10 points. Some points are contained in 3 planes from the configuration and the other points are contained in 5 planes. How many points are contained in 3 planes?
12. In the sequence of 2023 nonzero numbers, every number (except the first and the last one) is the product of its left and right neighbors. The fourth number is 5. Find the product of all numbers.
13. In the right triangle ABC with right angle B the legs have lengths $AB = 6$ and $BC = 8$. A circle passes through B and contains midpoints D , E , F of the sides AB , BC , AC . This circle intersects AC on one more point G , different from F . Find AG .
14. For positive x and y , among the inequalities $x + 2y \geq 9$, $2x + 3y \leq 16$, $2x + 5y \leq 20$, and $x \geq 11$, three are correct and one is wrong. Find the sum of all possible values of x .
15. Three circles are tangent to the coordinate axis Ox at the points 0, $1/3$, and 1. Also, the three circles are tangent to each other. Find the radius of the circle that is tangent to the coordinate axis at $1/3$.
16. The UPS truck had eight boxes with weights of 6, 11, 13, 14, 16, 20, 27, and 30 lbs. Ann and her neighbor Ben both received deliveries. The combined weight of Ann's delivery was three times greater than the combined weight of Ben's delivery. There was only one box left in the UPS truck. Which box was it?

17. What is the sum of all possible values of c such that there exists y for which the equation $|x^2 + c|x| + 1| = y$ has exactly 5 solutions in x .
18. How many pairs (x, y) of positive integers exist such that $\sqrt{x} + \sqrt{y} = \sqrt{2023}$?
19. The roots of the two polynomials $x^2 - ax + b$ and $x^2 - cx + d$ are four different positive integers. The coefficients a, b, c, d up to a permutation are four consecutive positive integers $n, n + 1, n + 2, n + 3$. Find the product of all possible values of n .
20. Right triangles $ABC, ACD, ADE,$ and AEF have right angles $B, C, D,$ and $E,$ respectively. These triangles are located outside of each other and $\angle BAC = \angle CAD = \angle DAE = \angle EAF = 30^\circ$. Find the angle BDF .