

Riesz's Pieces

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A sampling set is a set of points on an interval along with a reconstruction algorithm which uniquely determines functions in a Hilbert function space when they are evaluated at the points in the sampling set, and allows the function to be recovered from its values at the points. Since evaluation of a function at a prescribed point qualifies as a linear functional on the function space, the Riesz Representation Theorem states that there is also a vector in the space whose inner product with the function is equal to the evaluation of the function at the point. If the set of Riesz vectors corresponding to each sampling point has the additional property that they form a tight frame on the Hilbert Space, then the sampling set is called a Tight Sampling Set. We studied the space of polynomials on $[0,1]$, and found that the conditions for a Tight Sampling Set are rarely encountered with ordinary Riemann integration, but when a measure is introduced in the form of a density function on $[0,1]$, the equations derived provide values for the moments of the density function, leading to a special case of the Hausdorff Inverse Moment Problem.