Structural Properties of Wavelet Sets

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Abstract

A *wavelet set* is defined to be a measurable subset E of $\mathbb{R}$ for which $\frac{1}{\sqrt{2\pi}}\chi_E$ is the fourier transform of a wavelet. In one-dimension, a wavelet set is a 2π-translation and 2-dilation generator of a partition of the real line. We also define a *joint dilation wavelet set* as a wavelet set under dilation by both d_1 and d_2 for $d_1 \neq d_2$. We have proven that a set W is a joint (2) and (-2) dilation wavelet set if and only if $\bigcup_{j\in\mathbb{Z}} 4^j W$ is symmetric. By applying this result to some families of 2-dilation wavelet sets given by David Larson and Xingde Dai in their AMS memoir, we have proven that all of the Journe sets are joint (2) and (-2) dilation wavelet sets. Using the same tool, we can also find all 2-interval and 3-interval joint dilation wavelet sets. A characterization of all 3-interval wavelet sets different from the literature is included for our purpose.

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