

Extremal Questions for Tetranomials

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Abstract:

We present the first known examples of 2×2 fewnomial systems of type $(3,4)$ with 7 roots in the positive quadrant. It is easy to construct such systems with 6 roots in the positive quadrant, and Li, Rojas, and Wang have shown that these systems can never have more than 14 roots in the positive quadrant. Kushnirenko's Conjecture suggested that 2×2 fewnomial systems of type $(3,m)$ ($m > 2$) could have no more than $2m-2$ roots in the positive quadrant. While a counterexample to this conjecture with 5 roots was found by Haas for $m=3$, no counterexamples have previously been found for greater values of m . We also present an inductive construction for 2×2 fewnomial systems of type $(3,m)$ with $2m-1$ roots in the positive quadrant. This disproves Kushnirenko's Conjecture for all systems of type $(3,m)$ for $m > 2$.