

# Trojan Y-Chromosome Approach to Dealing with Invasive Species

Jared Brown  
St. Olaf College  
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# The Problem

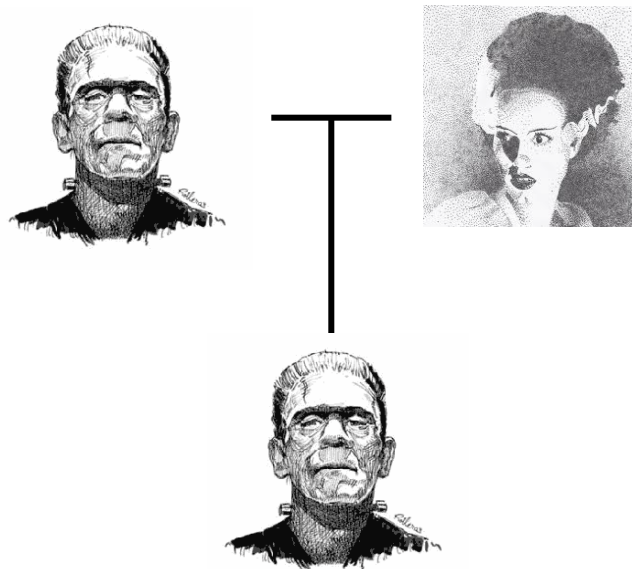
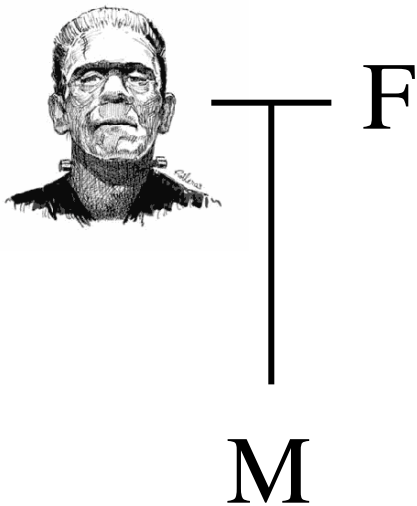
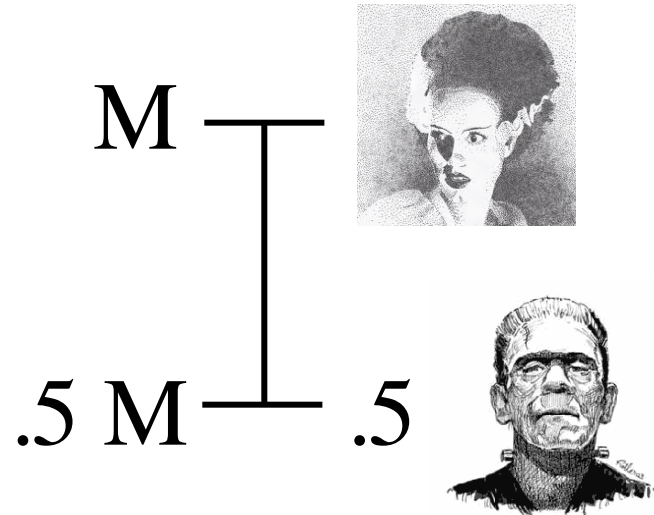
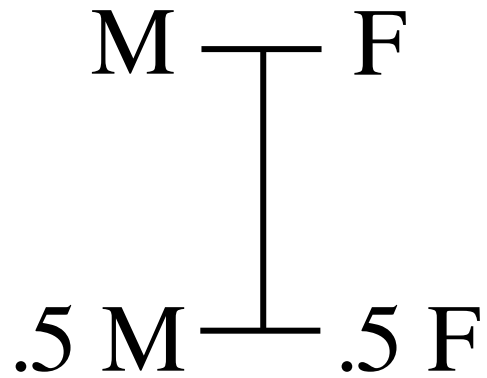


# Solutions

- Poison
- Introduction of Foreign Species



# Trojan Breeding



# Basic Model

$$\frac{df}{dt} = \frac{1}{2}\beta fmL - \delta f$$

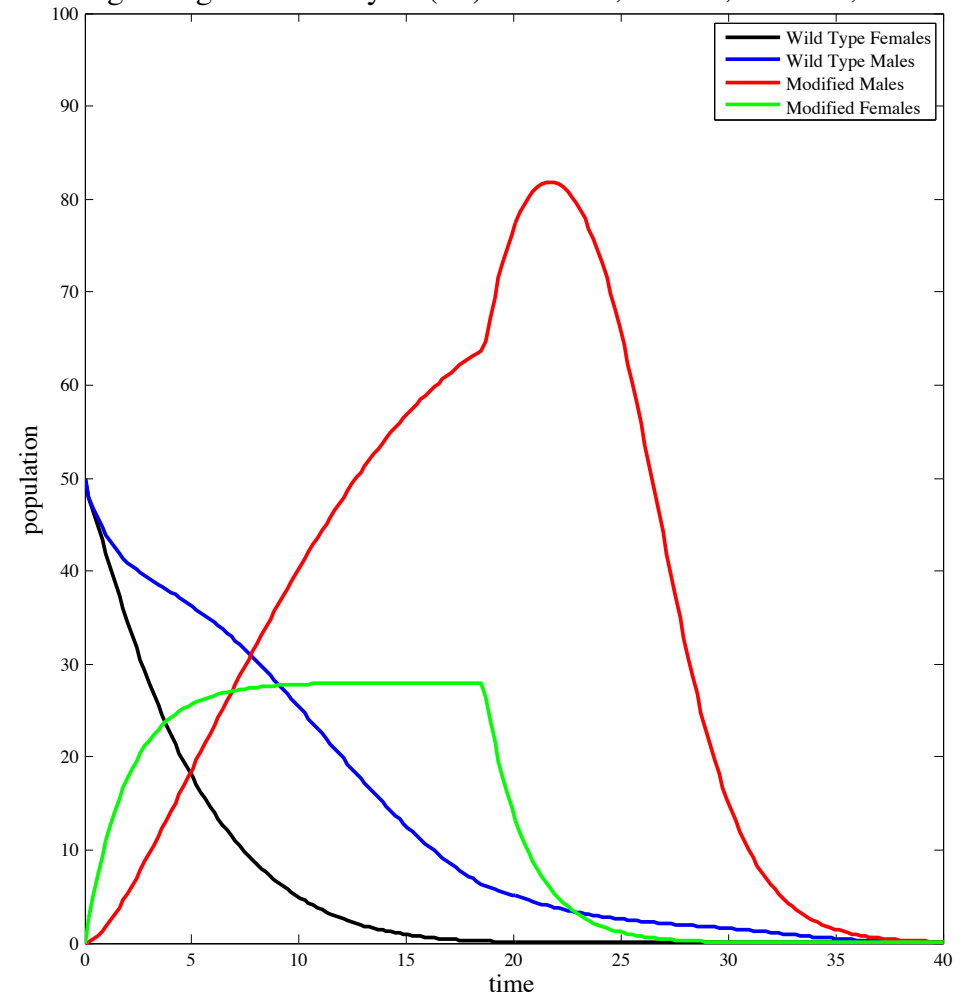
$$\frac{dm}{dt} = \left(\frac{1}{2}fm + \frac{1}{2}rm + fs\right)\beta L - \delta m$$

$$\frac{ds}{dt} = \left(\frac{1}{2}rm + rs\right)\beta L - \delta s$$

$$\frac{dr}{dt} = \mu - \delta r$$

$$L = 1 - \frac{f+m+s+r}{K}$$

Bang-Bang Basic Analysis (F4) for  $B = 1$ ,  $d = 0.5$ ,  $K = 100$ ,  $\mu = 14$



# Proportional Model

$$\frac{df}{dt} = \frac{1}{2}\beta fmP_{fm}L - \delta f$$

$$\frac{dm}{dt} = \left(\frac{1}{2}fmP_{fm} + \frac{1}{2}rmP_{rm} + fsP_{fs}\right)\beta L - \delta m$$

$$\frac{ds}{dt} = \left(\frac{1}{2}rmP_{rm} + rsP_{rs}\right)\beta L - \delta s$$

$$\frac{dr}{dt} = \mu - \delta r$$

$$L = 1 - \frac{f+m+s+r}{K}$$

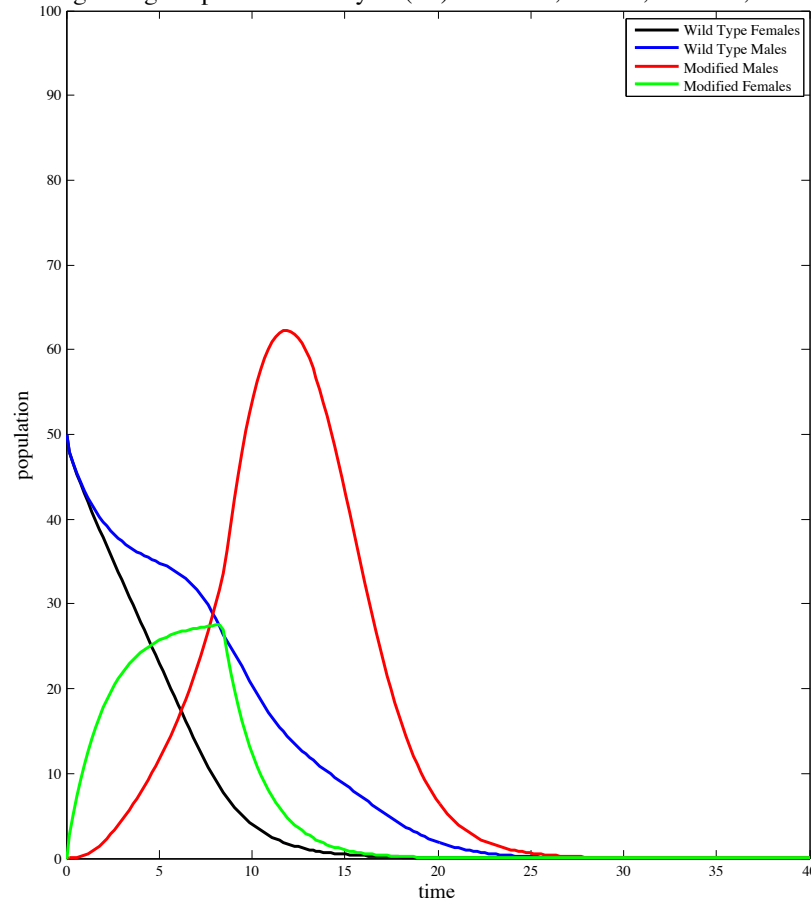
$$P_{fm} = \frac{fm}{(m+s)(f+r)}$$

$$P_{fs} = \frac{fs}{(m+s)(f+r)}$$

$$P_{rm} = \frac{rm}{(m+s)(f+r)}$$

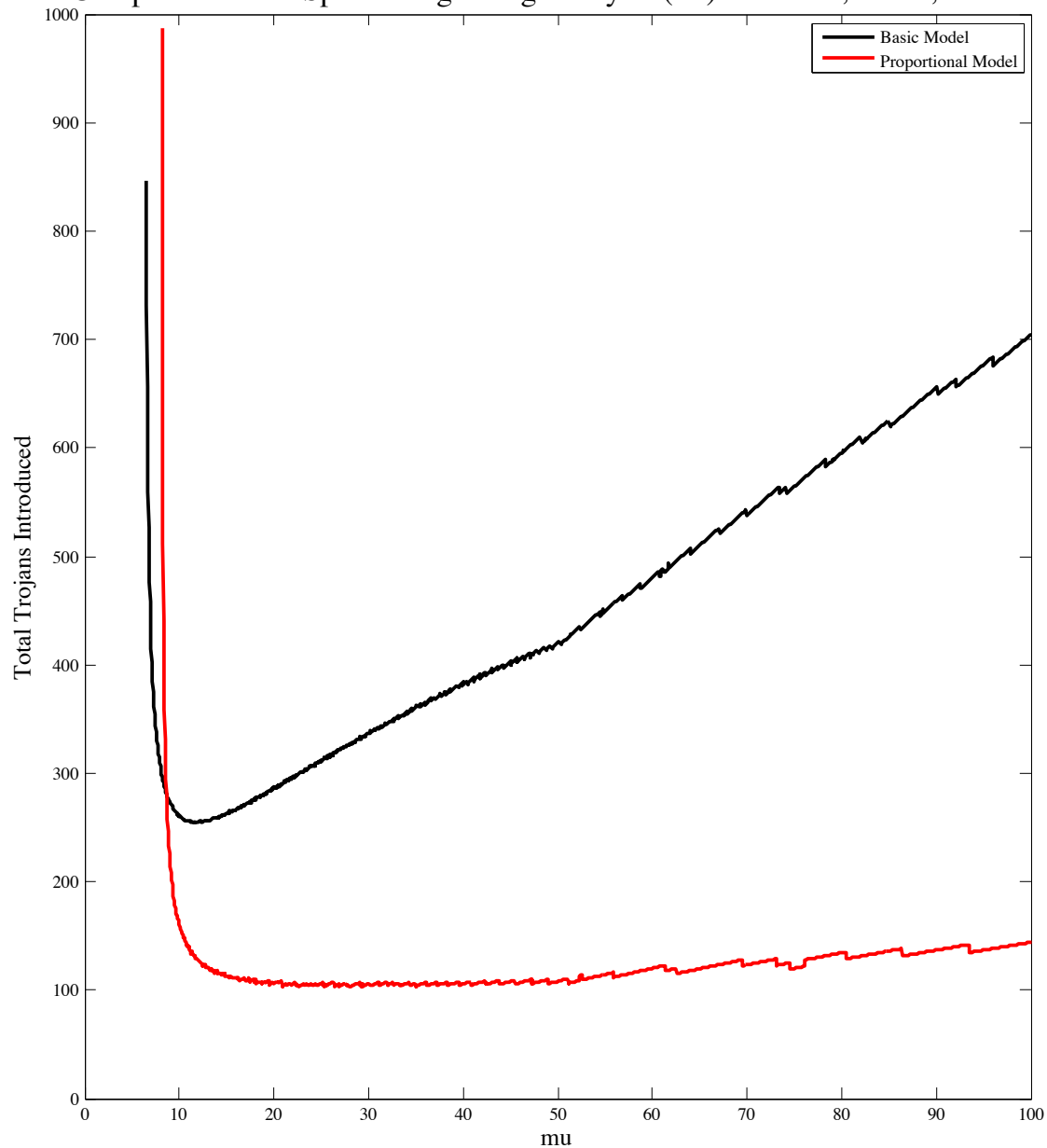
$$P_{rs} = \frac{rs}{(m+s)(f+r)}$$

Bang-Bang Proportional Analysis (F4) for B = 1, d = 0.5, K = 100, mu = 14



# $\mu$ Minimization

Comparative Mu-Space Bang-Bang Analysis (F4) for  $B = 1$ ,  $d = .5$ ,  $K = 100$



# Stochasticity

- Low Population Systems
- Random Events



# 0-Dimensional System

$$F \text{ Birth} = \frac{1}{2}\beta fm$$

$$M \text{ Birth} = \beta(\frac{1}{2}fm + \frac{1}{2}rm + fs)$$

$$S \text{ Birth} = \beta(\frac{1}{2}rm + rs)$$

$$R \text{ Birth} = \mu$$

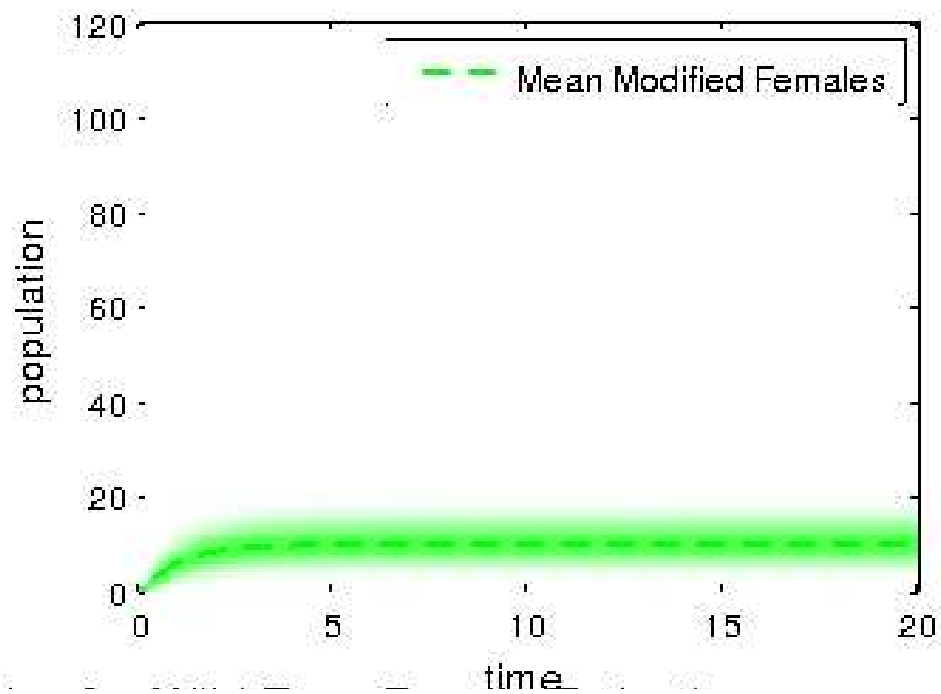
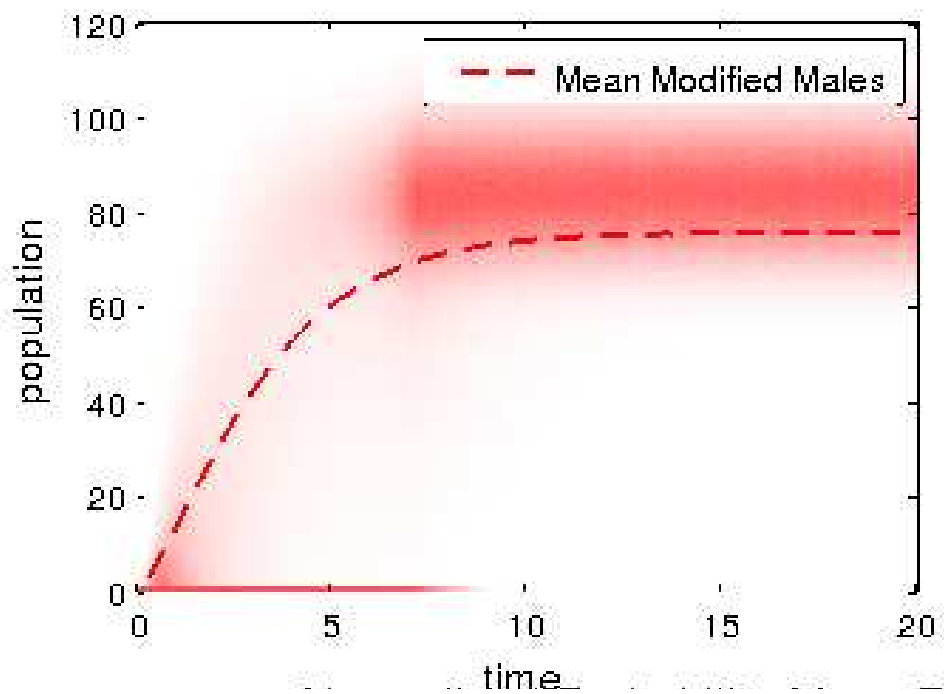
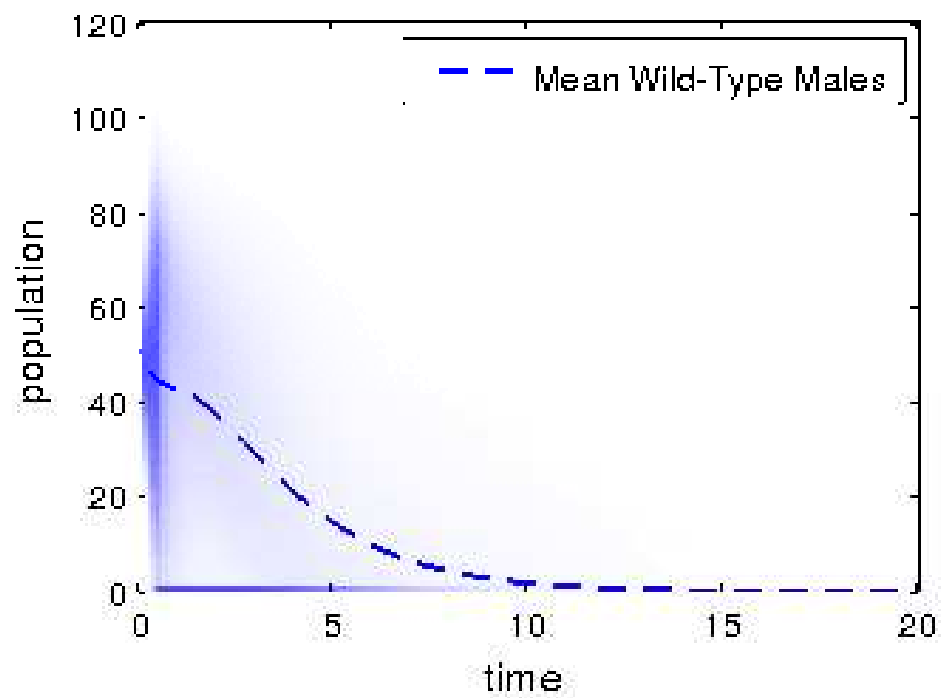
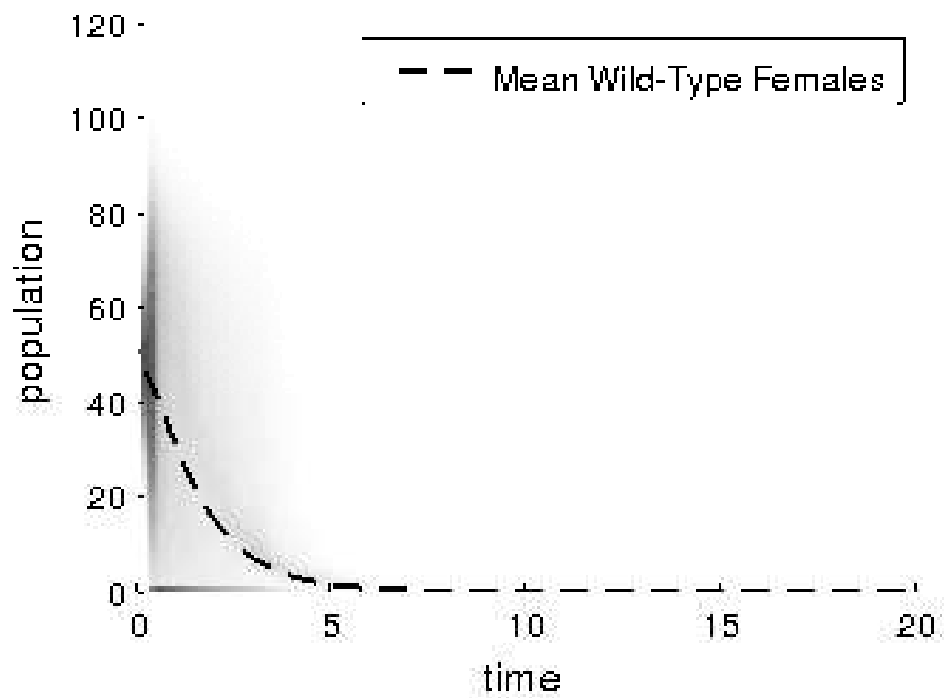
$$L' = \frac{f+m+s+r}{K}$$

$$F \text{ Death} = \frac{1}{2}\beta fmL' + f$$

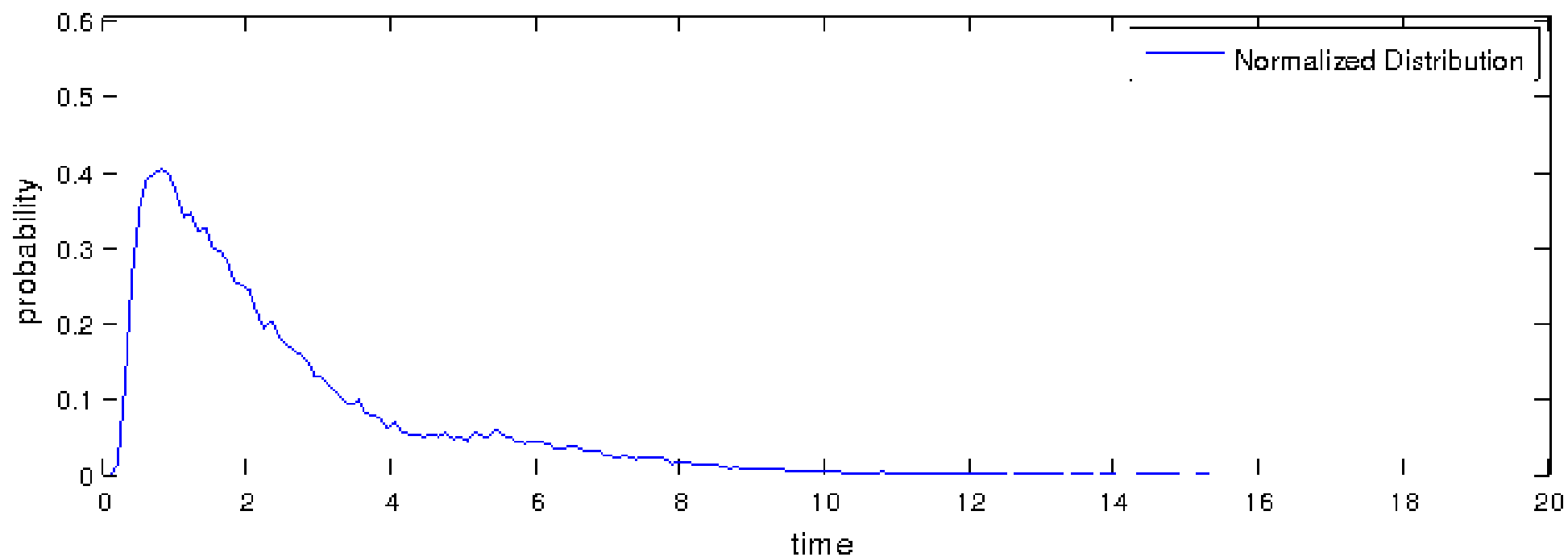
$$M \text{ Death} = \beta L'(\frac{1}{2}fm + \frac{1}{2}rm + fs) + m$$

$$S \text{ Death} = \beta L'(\frac{1}{2}rm + rs) + s$$

$$R \text{ Death} = r$$



Normalized Probability Mass Function for Wild-Type Female Extinction  
For mean = 2.4208, variance = 3.9316, and skewness = 12.1638



# 2-dimensional System

$$F \text{ Birth} = \frac{1}{2}\beta fm$$

$$M \text{ Birth} = \beta(\frac{1}{2}fm + \frac{1}{2}rm + fs)$$

$$S \text{ Birth} = \beta(\frac{1}{2}rm + rs)$$

$$R \text{ Birth} = \mu$$

$$L' = \frac{f+m+s+r}{K}$$

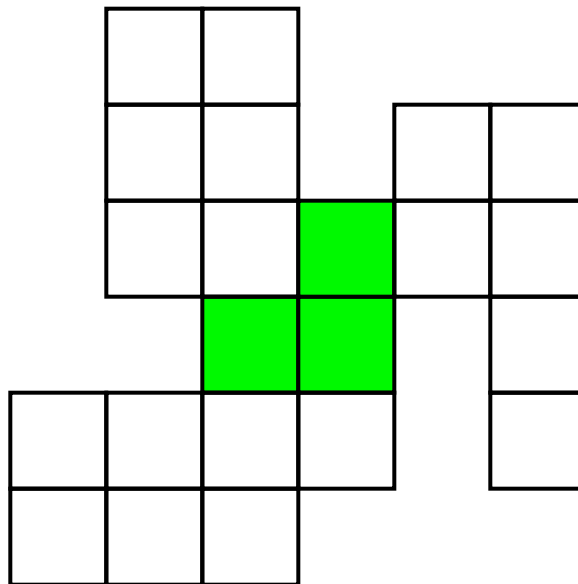
$$F \text{ Death} = \frac{1}{2}\beta fmL' + f$$

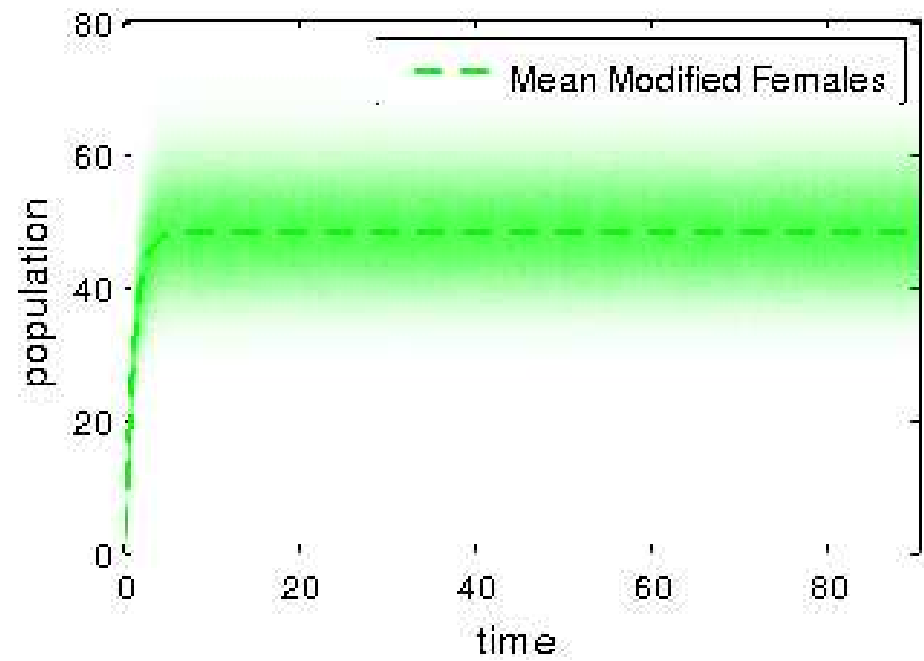
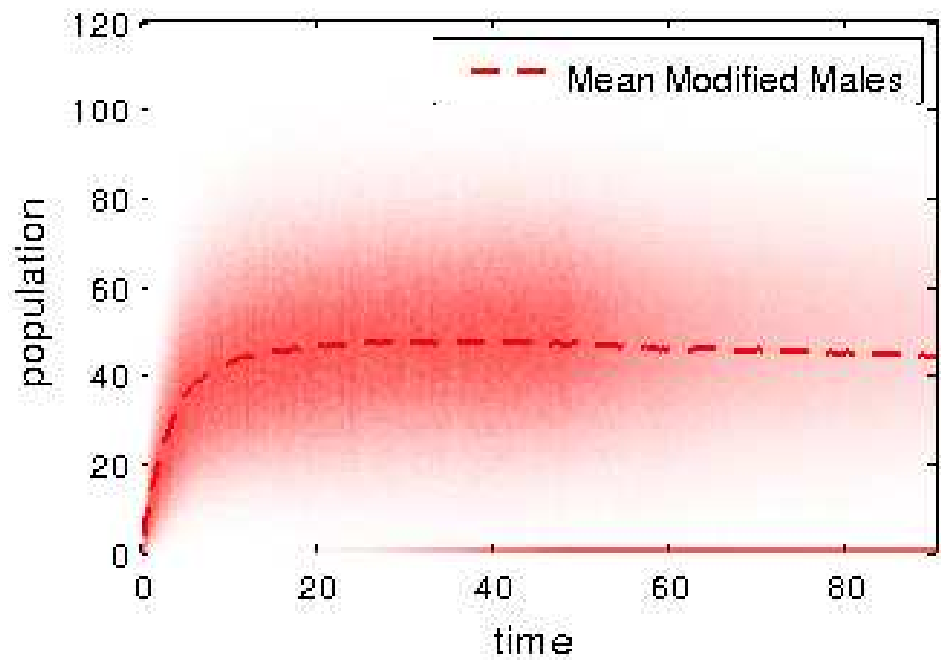
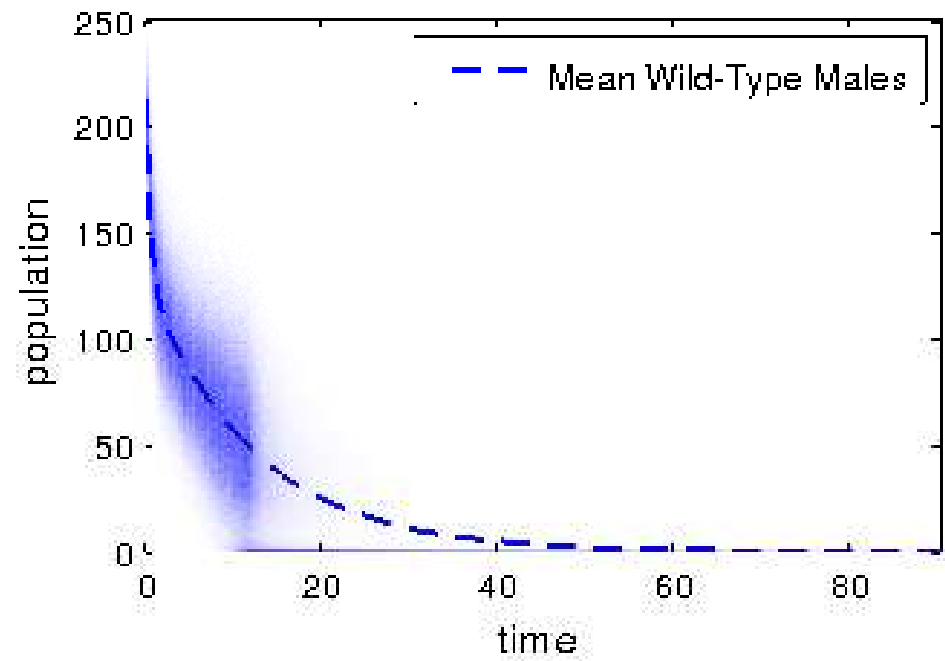
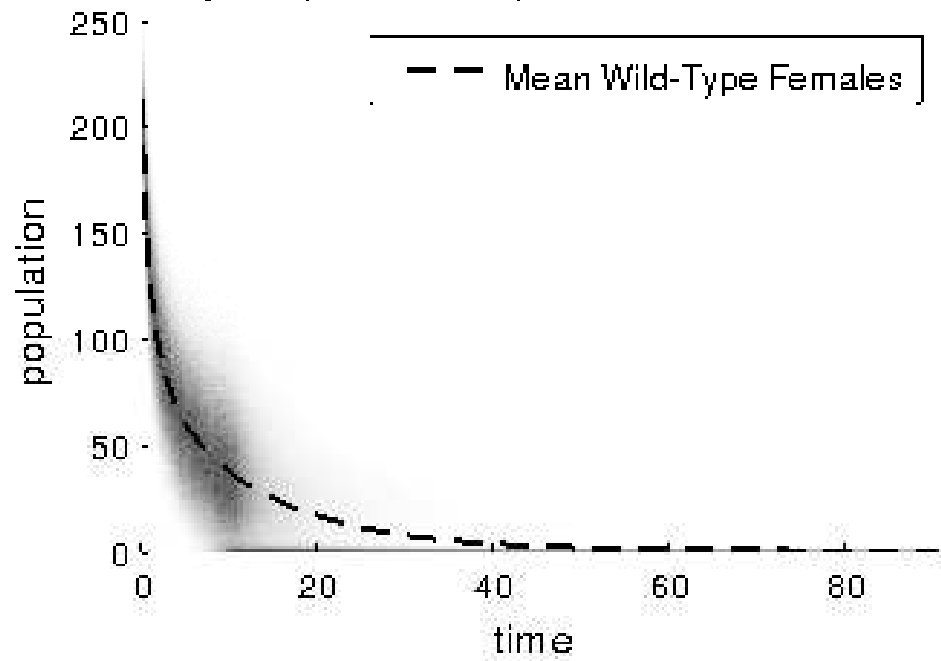
$$M \text{ Death} = \beta L'(\frac{1}{2}fm + \frac{1}{2}rm + fs) + m$$

$$S \text{ Death} = \beta L'(\frac{1}{2}rm + rs) + s$$

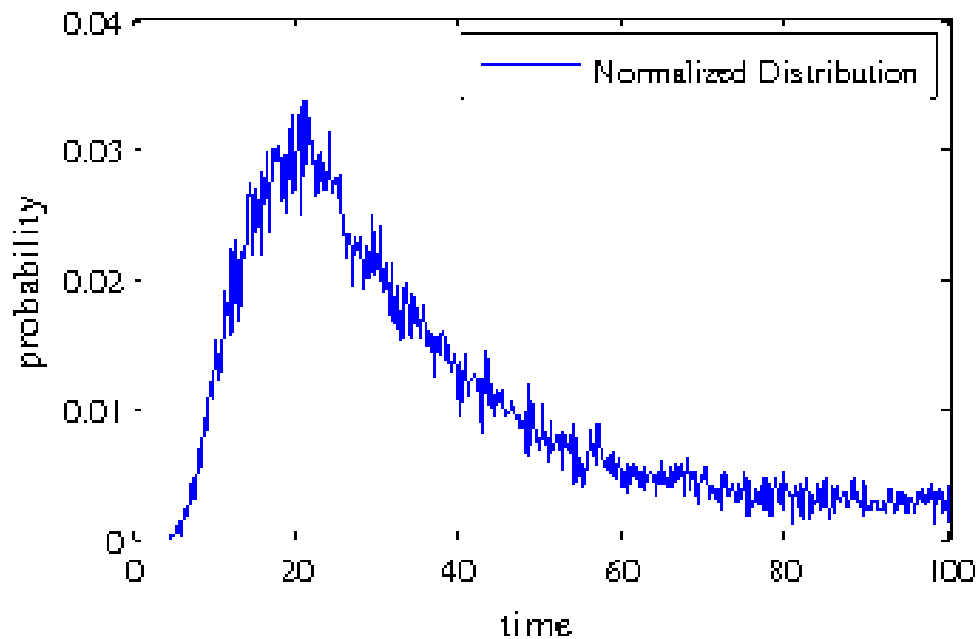
$$R \text{ Death} = r$$

$$\text{Migrate} = \frac{\text{population} * \text{migration\_speed}}{\text{cell\_length}}$$

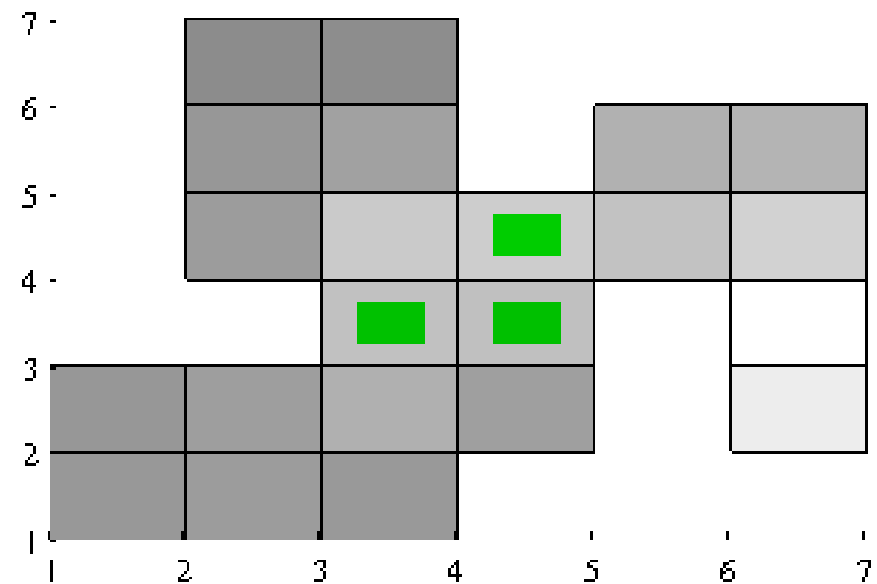




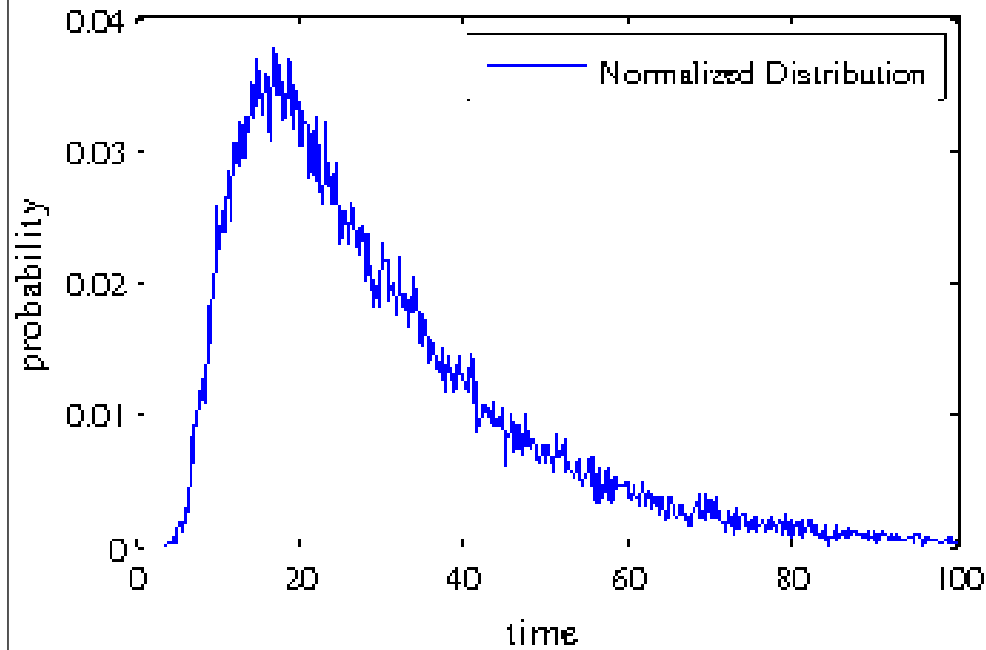
- mean = 35.3 (t $\delta$ )
- variance = 465
- skewness = 11,500



Max time = 27.7



- mean = 28.8 ( $t\delta$ )
- variance = 295
- skewness = 6690



Max time = 26.7

