Analyzing Methods to Determine Pairwise Correlations Between Neurons

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Okun *et al.* Experiment

- Neurons fire signals, known as *spikes*, in order to communicate with each other through an electrochemical process.
- Each neuron has a corresponding spike train, which is a sequence of spikes over time.
- In an attempt to explain relationships between neurons based on spike trains, Okun *et al.* discussed these complex individual neural activities and how they could be coordinated [3].
- Found that neighboring neurons were correlated based on the firings of the overall population and found that this provided a compact summary of the population activity.

RMM with Coupling Terms: The Parameters

3 parameters extracted to determine pairwise correlations:

1. Row sums **s**: number of spikes of a neuron
2. Column sums **c**: number of all spikes at $time = i$
3. Inner product of each row and **c**, **d**: stPR

Example

Neurons	**Raster Plot**								**s**	**d**
1	0	1	0	1	0	0	1	1	4	4
2	1	0	1	0	1	1	0	0	4	8
3	1	0	1	0	1	1	0	0	4	8
c	2	1	2	1	2	2	1	1		

- $M(s, c, d) = \{\text{set of matrices with prescribed } (s, c, d)\}$

RMM with Coupling Terms: Ryser and Spike Exchange

- First they generated a matrix satisfying **s** and **c** using Ryser's algorithm [1].
- Then the matrix was put in canonical form and a random spike exchange across neurons [2] was performed.

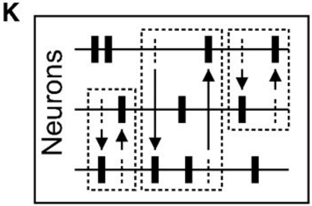


Figure: A representation of spike exchange across neurons [2]

RMM with Coupling Terms: Ryser and Spike Exchange

Example cont'd

Here is a possible matrix that we may obtain from this process:

Neurons	**New Raster Plot**								$\mathbf{s}^*$	$\mathbf{d}^*$
1	1	1	1	1	0	0	0	0	4	8
2	0	0	0	0	1	1	1	1	4	4
3	1	1	1	1	0	0	0	0	4	8
$\mathbf{c}^*$	2	2	2	2	1	1	1	1		

RMM with Coupling Terms: **d** Constraint

Example cont'd

Neurons	**New Raster Plot**								$\mathbf{s}^*$	$\mathbf{d}^*$	**d**
1	1	1	1	1	0	0	0	0	4	8	4
2	0	0	0	0	1	1	1	1	4	4	8
3	1	1	1	1	0	0	0	0	4	8	8
$\mathbf{c}^*$	2	2	2	2	1	1	1	1			

Exchange the 0's and the 1's in the boxed sub-matrix above:

Neurons	**New Raster Plot**								$\mathbf{s}^*$	$\mathbf{d}^*$	**d**
1	1	1	1	0	1	0	0	0	4	7	4
2	0	0	0	1	0	1	1	1	4	5	8
3	1	1	1	1	0	0	0	0	4	8	8
$\mathbf{c}^*$	2	2	2	2	1	1	1	1			

RMM with Coupling Terms: Pearson Correlation

Finally, the correlation between each pair of neurons from this new random matrix was computed using the Pearson correlation.

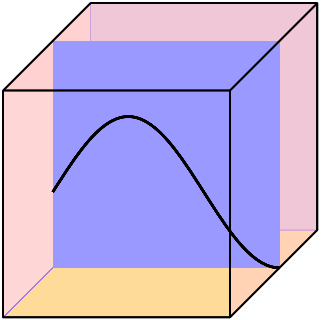
Definition

The *Pearson correlation* is a measure of strength of the linear relationship between two variables x and y.

$$r = \frac{\sum_{n=1}^{m}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{n=1}^{m}(x_i - \bar{x})^2}\sqrt{\sum_{n=1}^{m}(y_i - \bar{y})^2} \tag{1}$$

- -1 implies negative correlation
- 0 implies no correlation
- 1 implies positive correlation

Sampling Method: A Visualization



- Cube - $M(s, c)$
- Square - $M(s, c, d \pm n)$
- Curve - $M(s, c, d)$

Goals

Main Question

Will we be able to to obtain more accurate and consistent results if we do not allow for the $\pm n$ error on $\mathbf{d}$?

- Determine if the $\pm n$ error allowed on $\mathbf{d}$ made a difference in our correlation results or not.
- Create code that would perform the raster marginals model with coupling terms but without the error on $\mathbf{d}$.
- Create a program that would generate multiple sample matrices, both with and without the $\pm n$ on $\mathbf{d}$ and output their corresponding correlations.

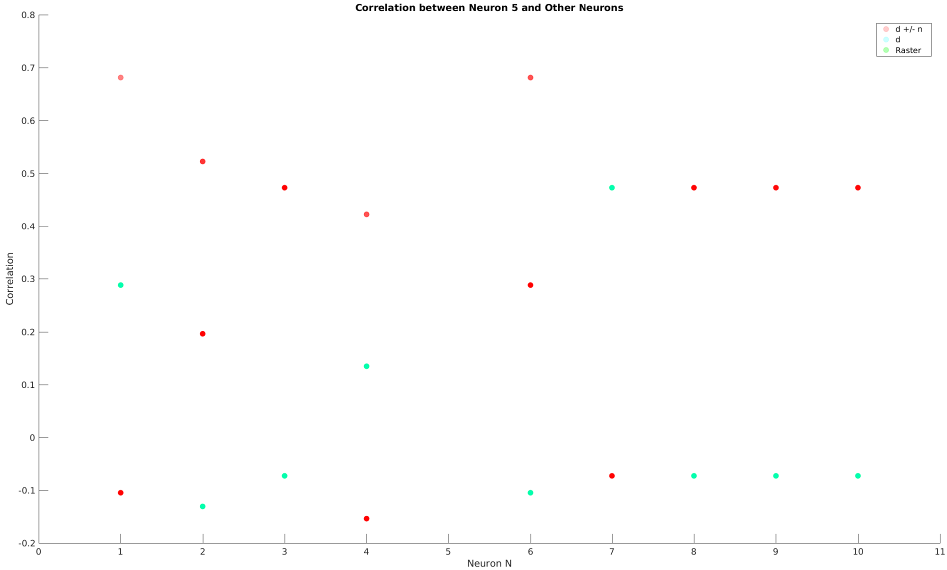
Modified Code & New Program

1. Inputs: raster file, number of columns, number of sample matrices from Dr. Okun's original code, and number of sample matrices from our modified code.
2. Generate a new raster file of a specified size.
3. Extract the three parameters and run Dr. Okun's code and our modified version of his code the specified number of times.
4. Calculate the correlation coefficient matrix.
5. Calculate the estimated standard deviations for both sample correlation values and compute the difference.
6. Plot the correlations.

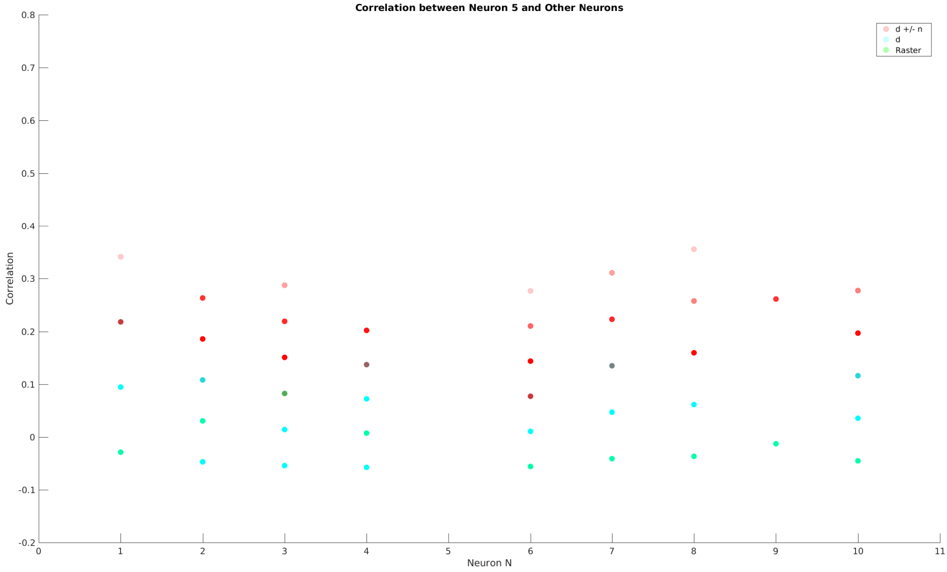
Explanation of Graphs

- Each run of the program produces n graphs, where each graph represents the correlation between neuron i and all other neurons.
- The x-axis represents the neurons and the y-axis represents the correlation.
- Here we use 100 samples each from Dr. Okun's code and our modified code.
  - Red dots - Dr. Okun's code
  - Blue dots - Modified code
  - Green dots - new raster file

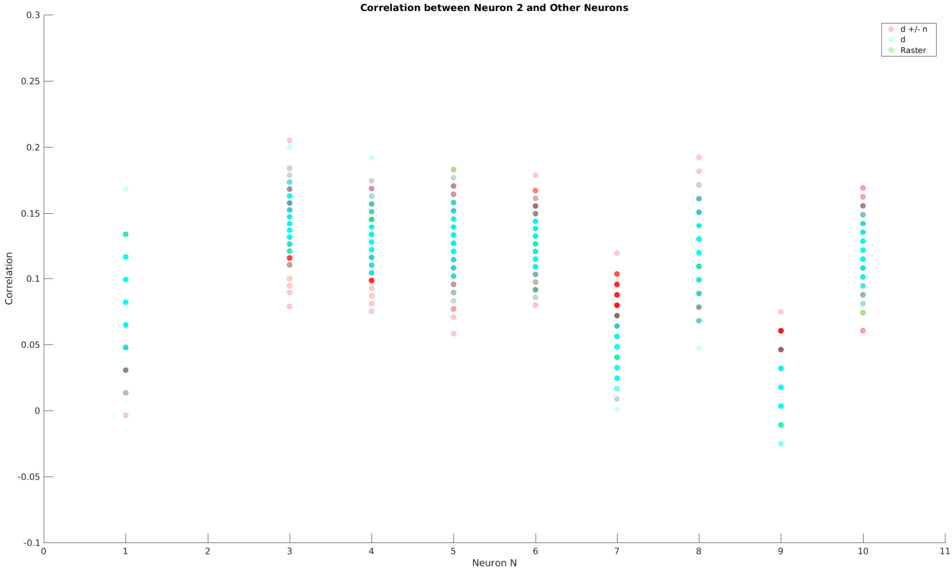
10×30 Raster Example



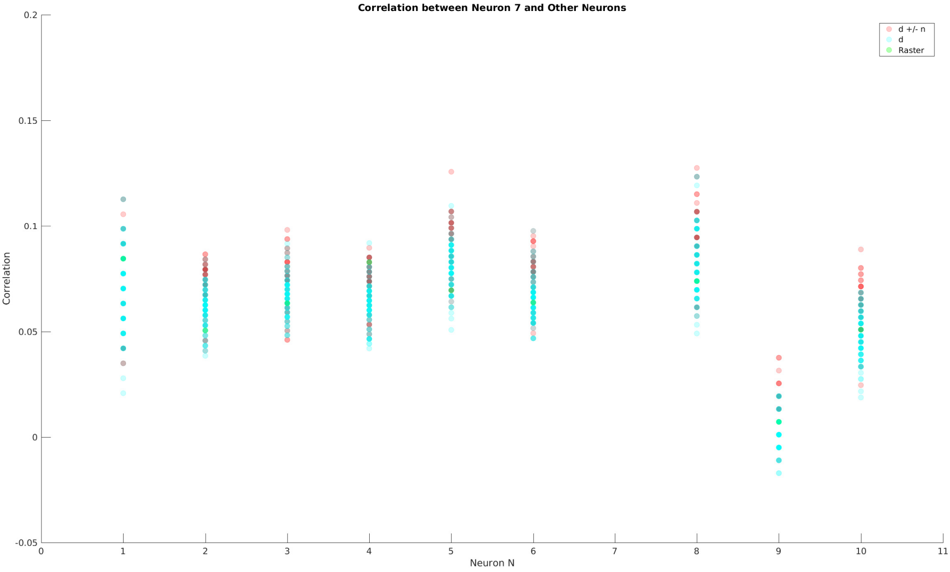
10 × 300 Raster Example



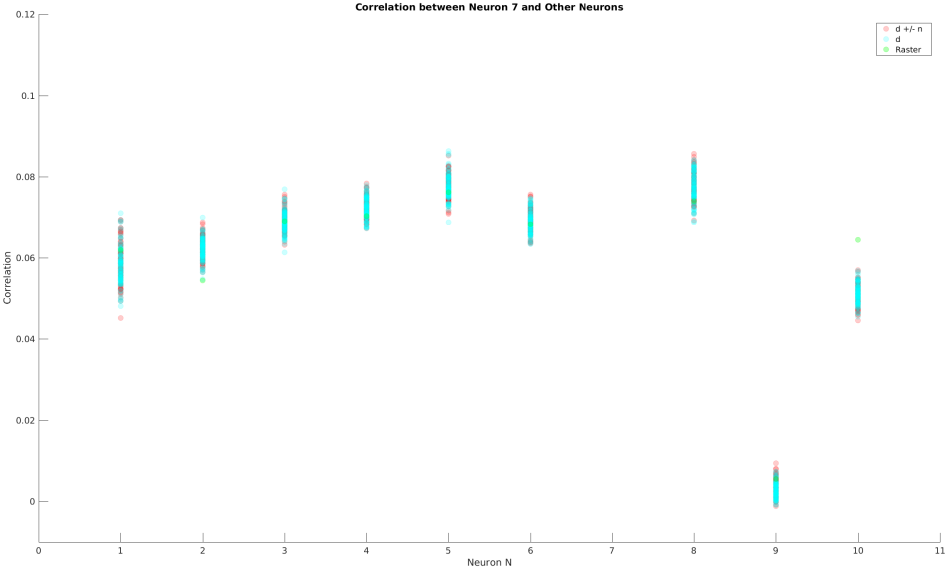
10 × 3000 Raster Example



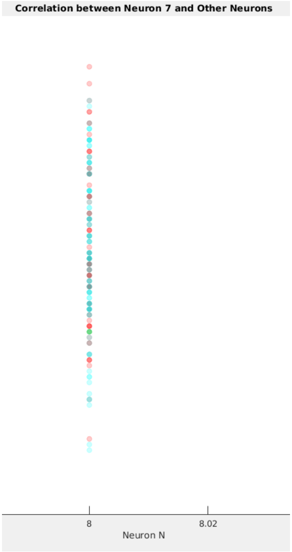
10 × 10000 Raster Example



10 × 170000 Raster Example



A Closer Look



What Does This Mean?

- As the number of columns increases, the tolerance on **d** matters less.
- As the number of columns increases, the more similar the standard deviations of both samples become.
- Losing the original permutation of **c** seems to have an effect on correlations.
- Important to note: these results and claims can only be applied to similar rasters.

Future Directions

- Test our claims and results on various rasters that we did not have access to.
- Determine if the provided three parameters are actually enough to determine pairwise correlations between pairs of neurons.
- Find bounds on the solution spaces for matrices with prescribed row sum, column sum, and inner product constraints.

Thank you

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References

[1] Richard A. Brualdi. Algorithms for constructing (0, 1)-matrices with prescribed row and column sum vectors. *Discrete Mathematics*, 306(23):3054-3062, 2006.

[2] Sonja Grün. Data-Driver Significance Estimation for Precise Spike Correlation. *J Neurophysiol*, 101(3):1126-1140, 2009.

[3] Michael Okun, Nicholas A. Steinmetz, Lee Cossell, M. Florencia Iacaruso, Ho Ko, Pter Barth, Tirin Moore, Sonja B. Hofer, Thomas D. Mrsic-Flogel, Matteo Carandini, and Kenneth D. Harris. Diverse coupling of neurons to populations in sensory cotex. *Nature*, 521(7553):511-515, 2015.