2019 Texas A&M REU Miniconference

July 22–23, Blocker Building, Room 628

SCHEDULE

MON., July 22

09:30–09:50	Asymptotic Distribution of the Partition Crank (Part 1)	Asimina Hamakiotes
09:55–10:15	Asymptotic Distribution of the Partition Crank (Part 2)	Aaron Kriegman
10:20–10:40	Preserving Identifiability: Removing Leaks, Moving Outputs, and a New Model	Seth Gerberding
10:45–10:55	*Break*	
10:55–11:15	Neural Bonanza (Part 1)	Brianna Gambacini
11:20–11:40	Neural Bonanza (Part 2)	Sam Macdonald
11:45–12:05	An Efficient Way of Understanding the Maximum Number of Steady States of Chemical Reaction Networks	Dilruba Sofia
12:00–13:00	Lunch in Blocker 627	

TUES., July 23

09:30–09:50	Probability of Easily Approximating the Positive Real Roots of Trinomials (Part 1)	Lauren Gernes
09:55–10:15	Probability of Easily Approximating the Positive Real Roots of Trinomials (Part 2)	Laurel Newman
10:20–10:40	On the Existence and Number of Roots of Polynomials over Prime Fields	Tyler Feemster
10:45–10:55	*Break*	
10:55–11:15	Counting Points on Arbitrary Curves Over Prime Power Rings	Caleb Robelle
11:20–11:40	An average of generalized Dedekind sums (Part 1)	Stephanie Gaston
11:45–12:05	An average of generalized Dedekind sums (Part 2)	Travis Dillon
12:00–13:00	Lunch in Blocker 627	

As always, we gratefully acknowledge the support of our Department Head Emil Straube, Associate Department Head Paulo Lima-Filho, Undergraduate Program Assistant Donna Hoffman, and the support of the National Science Foundation through REU grant DMS-1757872.

ABSTRACTS

(In order of appearance)

Asymptotic Distribution of the Partition Crank

Asimina Hamakiotes and Aaron Kriegman

The partition crank is a statistic on partitions introduced by Andrews-Garvan to explain Ramanujans congruences. In this talk, we prove the effective equidistribution of the asymptotic approximation of the distribution of the partition crank modulo Q, $M(r, Q; n) \to 1/Q$ as n gets large, by calculating the effective bounds on the error of M(r,Q;n). We use the effective bounds to prove subjectivity and strict log-subadditivity for the crank function.

Preserving Identifiability: Removing Leaks, Moving Outputs, and a New Model

Seth Gerberding

This talk addresses model identifiability, and we prove several cases where changing the model in certain ways preserves identifiability. We present a new model, the Fin model, and prove it is identifiable. Then, we show that removing certain edges from the Fin model, which generates what we term a Nemo model, preserves identifiability. We also prove that a conjecture from Gross, Meshkat, and Shiu (2019) holds for cycle, catenary, and mammillary models. Lastly, we examine a new kind of operation, moving the output, and show that it preserves identifiability in cycle models. Our proofs are aided by results on elementary symmetric polynomials and the theory of linear algebra for input-output equation of linear compartment models.

Neural Bonanza

Brianna Gambacini and Sam Macdonald

Neural codes are mathematical models of neural activity. Neuroscientists have discovered neurons called place cells which fire when animals are in specific (and usually convex) regions in space. Through monitoring these place cells and recording data on when they fire, we can construct neural codes, which tell us what neurons fire together. Of particular interest to the mathematical community is identifying which codes can be represented by open or closed convex sets. In our talks, we discuss the difference between open and closed convexity as well as other relevant definitions. We also provide counterexamples for two conjectures regarding closed convex neural codes along with several other results, including a new method for determining whether or not a code is open convex.

An Efficient Way of Understanding the Maximum Number of Steady States of Chemical Reaction Networks

Dilruba Sofia

Many chemical processes can be modeled mathematically by chemical reaction networks. A chemical reaction network can give us some ordinary differential equations that model the concentrations of the chemical species as time varies and conservation laws that constrain the total concentrations. Under mild hypotheses, this resulting system is a polynomial system. We can use this acquired system to solve for steady states of the network by setting the equations equal to zero and finding the values of the variables. For a generic polynomial system, the number of nonzero complex solutions is called the mixed volume. Since, the concentrations of chemicals can only be described as positive real numbers, we are interested in the instances when the mixed volume equals to the maximum number of positive real solutions, because this would allow us to efficiently compute the maximum number of steady states of a network. In this paper, we classify the monomolecular networks whose maximum number of steady states is equal to the mixed volume of the corresponding polynomial system and give examples of networks with more species molecules that have the same property.

Probability of Easily Approximating the Positive Real Roots of Trinomials

Lauren Gernes and Laurel Newman

Polynomial system solving lends itself to a variety of fields, including chemical reaction networks, geolocation, and semi-definite programming. However, calculating the positive real roots for generic trinomials is inefficient. More easily calculated polytopes such as the positive tropical variety are powerful tools for approximating these roots. While the positive tropical variety is known to frequently be isotopic to the positive zero set of a trinomial, under certain conditions it is not. Knowing the probability with which these conditions are met for trinomials gives us insight into the reliability of the positive tropical variety as an approximation tool for the positive zero set. In this talk, we discuss the probability that the positive tropical variety is isotopic to the positive zero set for trinomials with normally distributed coefficients.

On the Existence and Number of Roots of Polynomials over Prime Fields

Tyler Feemster

The study of prime fields has many important applications to cryptography, and due to their finite nature, all endomorphisms of prime fields can be written as a polynomial. However, root finding for arbitrary polynomials over prime fields is very difficult since we do not have full access to the technology of real and complex analysis. In my talk, I discuss some classical results on the number of roots of certain multivariate polynomials over prime fields. Then, I present some results on and algorithms for determining if roots exist and how to find them for specific types of multivariate and univariate polynomials.

Counting Points on Arbitrary Curves Over Prime Power Rings

Caleb Robelle

Counting points on algebraic curves over finite fields has applications in cryptography, integer factorization, and coding theory. These applications depend on the ability to quickly determine the number of points on an algebraic curve over some finite field, both chosen to have certain properties. Less is known about efficient algorithms for point counting over prime power rings, particularly for singular curves, where Hensel lifting becomes much more complicated and resolution of singularities becomes too costly. We give the first algorithm to count points on algebraic curves over $\mathbb{Z}/\langle p^k \rangle$ with complexity polynomial in $k\ log(p)$. We also provide a new, simpler proof of rationality of Igusa zeta functions for algebraic curves. This is joint work with Yuyu Zhu and J. Maurice Rojas.

An average of generalized Dedekind sums

Stephanie Gaston and Travis Dillon

Classical Dedekind sums have been extensively studied in a wide variety of contexts, including modular forms, topology, combinatorial geometry, and quadratic reciprocity. Several authors have introduced generalizations of the Dedekind sum that incorporate Dirichlet characters. We focus on one recent generalization, presenting an exact formula for its second moment which extends a classical result of Walum. Further, we derive bounds for the second moment.