Homework 10

Math 171H (section 201), Fall 2023

This homework is due on **Tuesday, October 24** at the start of class. (Turn in answers to questions 1–7.)

0. *(This problem is not to be turned in.)* Read Section 4.1
1. Find all global extrema of the function $f(x) = \frac{x+1}{x^2+1}$ on the interval $[0, 10]$.
2. For each of the following, give an example of such a function (a sketch of the graph with extrema labeled is fine) OR explain briefly why no such function exists.
   (a) a function with 1 local maximum, 1 local minimum, and no global extrema
   (b) a function with 2 local minima, 1 global minimum, and no local maxima
   (c) a function with no local extrema
   (d) a function with infinitely many global extrema
   (e) a function with 3 global maxima and 2 global minima
3. Let $f(x)$ be a continuous function with domain a closed interval $[a, b]$. Prove that if $f(x)$ has 3 local maxima, then $f(x)$ has (at least) 2 local minima.
4. Complete the following claim and then prove it: *$f(x)$ has a local maximum at $x = a$ if and only if $-f(x)$ has a __________ at $x = a$.*
5. (a) Prove the following claim: *If $f(x)$ is a continuous function on a closed interval $[a, b]$, then there is a positive number N such that $-N < f(x) < N$ whenever x is in $[a, b]$.*
   (b) Is the claim in (a) true if the closed interval is replaced by an open interval? If yes, prove it; if not, disprove via a counterexample.
6. How are the critical numbers of a differentiable function $f(x)$ and its square $f(x)^2$ related? Explain your answer.
7. Prove or disprove the following claims:
   (a) If $f(x)$ has a local extremum at $x = a$, then so does $|f(x)|$.
   (b) If $|f(x)|$ has a local extremum at $x = a$, then so does $f(x)$.