Homework 12

Math 171H (section 201), Fall 2023

This homework is due on **THURSDAY, Nov. 9** at the start of class. (Turn in answers to questions 1–5.)

0. Read Section 4.4–4.6

1. Consider the function $f(x) = 1 + 3x^2 - 2x^3$.

   (a) Use derivative tests to determine where f is increasing, decreasing, concave up, and concave down.

   (b) Are there any inflection points?

   (c) Sketch a graph based on your above answers.

2. Prove the following: *If f and g are continuous and $[a, b]$ and differentiable on (a, b), then there is a number c in (a, b) such that*

   $$(f(b) - f(a))\, g'(c) \;=\; (g(b) - g(a))\, f'(c)\ .$$

   HINT: Apply a theorem to the following function:

   $$(f(x) - f(a))\,(g(b) - g(a)) - (f(b) - f(a))\,(g(x) - g(a))\ .$$

3. Compute the following limit:

   $$\lim_{x\to\infty} \left(\frac{3x-1}{2-x}\right) + \left(1 + \frac{5}{x}\right)^x\ .$$

4. For each of the following functions, find *all* local extrema (max or min) and *all* global extrema. (*Hint*: ideas from the next problem might be useful. Also, you can always check your answer using a graphing calculator.)

   (a) $2x^3 - 3x^2$

   (b) $\frac{1}{3}x^3 + 4x$

   (c) $e^x + \sin x$ with domain $[0, \infty)$

   (d) $e^x + x^x$ with domain $(1, \infty)$

5. These problems pertain to the *discriminant* of a quadratic equation. You can review this topic on page 13 at `https://people.tamu.edu/~j-epstein/Math147/Chapter1.1.pdf`.

   (a) Does $x^2 - 5x + 2 = 0$ have a real solution? Explain.

   (b) Does $x^2 - 2x + 5 = 0$ have a real solution? Explain.

   (c) Does $x^2 + 4 = 0$ have a real solution? Explain.

   (d) Is $f(x) = x^2 - 6x + 1$ always positive? Explain.

   (e) Is $f(x) = -x^2 + x + 6$ always negative? Explain.