

# Homework 4

Math 171H (section 201), Fall 2023

This homework is due on **Tuesday, Sept. 12** at the start of class. (Turn in answers to questions 1–7.)

0. Read Sections 2.2–2.5.
1. Draw the graph of a function  $f(x)$  that satisfies all of the following properties simultaneously:
  - (a)  $f(x)$  is continuous at  $x = 0$
  - (b)  $f(1) = 2$  and  $\lim_{x \rightarrow 1} f(x) = 3$
  - (c)  $\lim_{x \rightarrow 2} f(x)$  exists, but  $f(x)$  is not defined at  $x = 2$
  - (d)  $\lim_{x \rightarrow 3^-} f(x)$  does not exist, but  $f(x)$  is continuous from the right at  $x = 3$
2. Prove limit laws #7–8 (from class).
3. Prove 2 of the limit laws #1–6 (among the ones we did *not* prove in class).
4. Let  $m, b, a$  be real numbers. Let  $f(x) = mx + b$ .
  - (a) Determine the limit  $\lim_{x \rightarrow a} f(x)$ .
  - (b) Prove your answer to (a) using the definition of limit (using  $\epsilon$  and  $\delta$ ).
  - (c) Prove your answer to (a) using limit laws.
5. Let  $n$  be a positive integer. Let  $f(x) = x^n$ .
  - (a) Determine the limit  $\lim_{x \rightarrow 0} f(x)$ .
  - (b) Prove your answer to (a) using the definition of limit (using  $\epsilon$  and  $\delta$ ).
  - (c) Prove your answer to (a) using limit laws.
6. Compute the following limits – or state that the limit does not exist (no proofs needed):

(a)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 12}{x + 3}$$

(b)

$$\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x + 3}$$

(c)

$$\lim_{x \rightarrow -1^-} \frac{x^2 + 5}{x + 1}$$

(d)

$$\lim_{x \rightarrow -1} \frac{x^2 + 5}{x + 1}$$

7. For each function below, determine the value(s) of  $a$  for which  $f(x)$  has a limit at  $x = 0$ . (No proofs needed, but show your work.)

(a)

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x + a & \text{if } x > 0 \end{cases}$$

(b)

$$f(x) = \begin{cases} x + a & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

(c)

$$f(x) = \begin{cases} 2 & \text{if } x \leq 0 \\ (x - 1)^2 + a & \text{if } x > 0 \end{cases}$$

8. (Optional)

(a) Determine your learning style(s) through the following quiz:

<https://vark-learn.com/the-vark-questionnaire/>

(b) Find study strategies for your learning style(s) here:

<https://vark-learn.com/strategies/>