

Homework 4

Math 220 (section 906), Fall 2018

This homework is due on Thursday, September 20. You may cite results from class, as appropriate.

0. (*This problem is not to be turned in.*) Read Sections 2.1, 2.2, and 3.1.
 - (a) What is the difference between a proof by contradiction and a proof by contrapositive?
 - (b) What is the Fundamental Theorem of Algebra?
 - (c) Prove that an integer n is even if and only if $-n$ is even.
 - (d) Conclude (explain why you can!) that an integer n is odd if and only if $-n$ is odd.
 - (e) Prove that an integer n is even if and only if its last digit (the ones digit) is 0, 2, 4, 6, or 8. (*Hint:* For $n > 0$, consider the remainder after dividing by 10; for $n < 0$, use a previous problem.)
 - (f) Conclude (explain why you can!) that an integer n is odd if and only if the last digit is 1, 3, 5, 7, or 9.
1. Read Francis Su's *Guidelines for good mathematics writing*¹.
 - (a) List one thing from this document that was surprising or interesting to you.
 - (b) Pick one proof from Section 2.1, 2.2, or 3.1 in your book, and analyze it with respect to Su's advice. Where do you see advice being followed (or not)?
2. Prove the following claim (we outlined the proof in class): *For every integer n , if $2|n^2 - 1$, then $4|n^2 - 1$.*
3. Prove or disprove the following claims:
 - (a) There is a smallest positive integer.
 - (b) There is a largest real number.
 - (c) For an integer a , the following holds: $a^2|a$ if and only if $a = 0$ or $a = 1$ or $a = -1$.
 - (d) For integers n , x , and y , if $n \nmid xy$, then $n \nmid x$ and $n \nmid y$.
 - (e) For integers x and y , if $3|x$ and $5|y$, then $8|(x + y)$.
 - (f) $(\forall a \in \mathbb{Z})(\forall b \in \mathbb{Z})(3|a \Rightarrow 9|(ab))$
 - (g) $(\exists x \in \mathbb{R}) \wedge (\exists y \in \mathbb{Z})s.t. x - y = -5$
 - (h) $(\exists a \in \mathbb{Z}) \wedge (\exists b \in \mathbb{Z})s.t. a - b = 0.5$
 - (i) For integers a and b , if $7a + 3b$ is even, then a and b are of the same parity.
 - (j) $\sqrt[3]{2}$ is irrational.
 - (k) If n is a positive integer, then $5|(9^n - 4^n)$.
4. Section 2.1 #5, 12
5. Section 2.2 #3, 9
6. Section 3.1 #2

¹Available here: <https://www.math.hmc.edu/~su/math131/good-math-writing.pdf>

Writing Assignment 3

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This homework is due on Thursday, September 27 (so, you have 2 weeks to complete this). *Complete this part on a separate piece of paper, not the same paper for Homework.*

1. List the sources you plan to use for your term paper (websites, articles, reference books, etc.)
2. Write one paragraph describing what you expect to be the main message of your paper.
3. List three key ideas that you expect to develop in your paper.