

Homework 10

Math 300, Fall 2022

This homework is due on Friday, October 28. (Turn in your answers to questions 1–4.)

0. (*This problem is not to be turned in.*) Read Sections 5.4–5.5.

(a) Section 5.4 #1,2

(b) Section 5.5 #1,2, 5, 8–11

(c) Assume that a, b, c, d are real numbers with $a \neq 0$ and $c \neq 0$. Let $f(x) = ax + b$ and $g(x) = cx + d$ be functions (with both domain and codomain equal to \mathbb{R}). Is the function $h := g \circ (f^{-1})$ invertible? Bijective? Explain.

(d) Consider the following function:

$$f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R} \\ (x, y) \mapsto (y, x + y)$$

Consider another function:

$$g : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R} \\ (u, v) \mapsto (v - u, u)$$

Compute the compositions $g \circ f$ and $f \circ g$. Is f bijective? Explain.

1. Consider the following function:

$$f : \mathbb{R} \setminus \left\{ \frac{3}{5} \right\} \rightarrow \mathbb{R} \setminus \left\{ \frac{2}{5} \right\} \\ x \mapsto \frac{2x + 1}{5x - 3} .$$

Is it invertible? Is it bijective? Prove your answers.

2. Is the following function $f : \mathbb{R} \rightarrow \mathbb{R}$ invertible? If so, find the inverse function:

$$f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 . \end{cases}$$

3. Section 5.4 #1(i), 2(a)

4. Section 5.5 #3, 4, 6