Homework 3

Math 302 (section 501), Fall 2016

This homework is due on Thursday, September 15.

0. *(This problem is not to be turned in.)*
   (a) Read Sections 2.3 (including floor, ceiling, and factorial functions) and 3.1.
   (b) Is it true that $3.5 \in [2, 6]$?
   (c) (Practice Problems) Section 2.3 #16, 22, 33b, 36, 37, 40, 73
   (d) (Practice Problems) Section 3.1 #3, 10, 16, 52, 56
1. List *all* sets X for which $\{3, 4\} \not\subseteq X \subseteq \{2, 3, 4\}$.
2. *Beginning proofs:* State an appropriate first sentence of a proof to ...
   (a) show that $A \subset B$ (where A and B are sets).
   (b) show that $f = g$ (where $f : X \to Y$ and $g : X \to Y$ are functions).
   (c) show that a function $f : A \to B$ is one-to-one.
3. *Critiquing proofs:* Imagine you are grading a student's homework. For each of the following excerpts from a proof, determine if there is an error. If so, what is the error (explain it), and what do you think the student meant to write (your best guess)?
   (a) $a \in A = b \in B$
   (b) $3.5 \subseteq [0, 10]$
   (c) $x \leq 10 \iff x < 10$
   (d) $[0, 5] \ \cap \ [5, 6] \ = \ 5$
4. Prove or disprove the following claims:
   (a) $[0, 5] \cap [4, 6] = [4, 5]$
   (b) $[0, 5] \cup [4, 6] = [4, 5]$
5. (a) What is the cardinality of the set of all functions with domain $\{1, 2\}$ and codomain $\{a, b, c\}$? Explain your answer.
   (b) What is the cardinality of the set of all *one-to-one* functions with domain $\{1, 2\}$ and codomain $\{a, b, c\}$? Explain.
6. Complete the following claim, and give a proof: *a function $f : A \to B$ is a bijection if and only if there exists a function $g : B \to A$ such that the composition $f \circ g$ is the identity function on the set ______ and $g \circ f$ is the identity function on ______.*

7. Determine whether the function $f : \mathbb{R} \to \mathbb{R}^+$, given by $f(x) := e^x$, is a bijection. (*Hint*: you may use the previous problem.)

8. Write an algorithm whose input is a function from $\{1, 2, \dots, n\}$ to $\{1, 2, \dots, m\}$, and whose output is 'yes' if and only if f is onto.

9. Section 2.3 #3, 8, 12, 13

10. Section 3.1 #2, 34, 38, 41, 42