

Homework 7

Math 302 (section 501), Fall 2016

This homework is due on Thursday, October 13.

0. (*This problem is not to be turned in.*)

(a) Read Section 1.7

(b) (Practice Problems) Section 1.6 # 11, 12, 19, 35

(c) (Practice Problems) Section 1.7 # 4, 10, 11, 13, 18, 19, 20, 22, 25

1. Re-read Section 1.6, and then describe, in words, the following rules: *existential instantiation*, *existential generalization*, and *universal modus ponens*.

2. The author of your textbook, on page 82, states that proofs in the textbook avoid the words “obviously” and “clearly” – and also avoid omitting too many steps. Why do you think the author made this decision, and why does he point it out?

3. Is there something wrong with this supposed proof? If so, identify all the errors, and then either prove or disprove the claim. If not, explain why the proof is complete.

Claim: The average of three even numbers is an even number.

Proof: We proceed by contradiction: assume that the average of three even numbers is odd. However, the average of 2, 4, and 6, which is 4, is even. This is a contradiction.

4. Is there something wrong with this supposed proof? If so, identify all the errors, and then either prove or disprove the claim. If not, explain why the proof is complete.

Claim: $|\{x \in \mathbb{R} \mid x^3 = x\}| = 2$.

Proof: If $x^3 = x$, then $x^2 = 1$ and hence $x = 1$ or $x = -1$. Thus, the cardinality of $\{x \in \mathbb{R} \mid x^3 = x\}$ is 2.

5. Is there something wrong with this supposed proof? If so, identify all the errors (for this problem only, you do not need to prove or disprove the claim). If not, explain why the proof is complete.

Claim: If $f : A \rightarrow B$ and $g : B \rightarrow C$ are both functions that are bijections, then $(g \circ f) : A \rightarrow C$ is also a bijection.

Proof: Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$ are both functions that are bijections. Then, by definition, both f and g are one-to-one and onto. Hence, $(g \circ f)$ is too. So, by definition, $(g \circ f)$ is a bijection.

6. Section 1.6 # 18, 20, 26

7. Section 1.7 # 2, 8, 9, 12, 42