Homework 8

Math 302 (section 501), Fall 2016

This homework is due on Thursday, October 20.

0. *(This problem is not to be turned in.)*
   (a) Read Sections 1.8 and 2.4.
   (b) Use quantifiers to define *one-to-one* and *onto*. Negate both.
   (c) What types of theorems require a *uniqueness proof* (pg. 99)? What are the parts of such a proof? Prove that there is a unique real number x for which $x^2+6x+9=0$.
   (d) (Practice Problems) Section 1.8 # 2, 3, 8, 13, 17, 39, 34
   (e) (Practice Problems) Section 2.4 # 4, 10, 12, 18, 26, 32, 34, 40
1. Consider the following function:
$$\begin{array}{rccl} f: & \mathbb{Z}\times\mathbb{Z}\times\mathbb{Z} & \to & \{n\in\mathbb{Z} \mid n\geq 0\} \\ & (a,b,c) & \mapsto & a^2+b^2+c^2 \end{array}$$
   (a) Prove or disprove: f *is one-to-one.*
   (b) Prove or disprove: f *is onto.*
2. Prove or disprove the following claims:
   (a) If x and y are real numbers, then
$$\max(x,y) \quad = \quad \frac{x+y+|x-y|}{2} \ .$$
   (b) If x is a nonnegative real number, then $x^2-3x+2\geq 0$.
3. Prove the following claims:
   (a) If x and y are real numbers, then $|x|-|y|\leq |x-y|$.
   (b) If w and z and real numbers with $0<w<z$, then
$$w \quad < \quad \sqrt{wz} \quad < \quad \frac{1}{2}(w+z) \quad < \quad z \ .$$
4. Let $S := \{1,2,4,7\}$. Compute the following sums (also write down the summands):
$$\sum_{j=0}^{4}(-2)^j, \quad \sum_{j\in S} j\cdot(j-1), \quad \sum_{i=0}^{2}\sum_{j=0}^{3}(3i+2j), \quad \sum_{i=0}^{2}\sum_{j=0}^{3} ij \ .$$
5. Prove that for every $n\in\mathbb{Z}^+$ and every $a_1,\ldots,a_n\in\mathbb{R}$ it holds that $\sum_{j=1}^{n}(a_j-a_{j-1}) = a_n-a_0$. Do not use an argument involving "$\cdots$". Instead, give a direct argument exploiting how often particular terms appear and which sign they have.
6. Section 1.8 # 9, 14
7. Section 2.4 # 2, 6, 14, 16