Homework 7

Math 415 (section 502), Fall 2015

This homework is due on Thursday, October 15. You may cite results from class or previous homeworks, as appropriate. Recall that the *kernel* of a homomorphism $\phi : G \to G'$ is $\ker(\phi) := \phi^{-1}[\{e_{G'}\}]$.

0. *(This problem is not to be turned in.)*
   (a) Read Section 11.
   (b) Section 11 # 10, 11, 13, 16, 18
   (c) Does S_7 have any cyclic subgroups of order 9?
   (d) What is the smallest n for which S_n contains a permutation of order 10? What about order 9?

1. (No proofs necessary for this problem.)
   (a) List all subgroups of $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_4$.
   (b) Let $g = (132) \in S_3$, and let H be the subgroup generated by (12) in S_3. Compute gH and Hg.

2. (a) Is $\mathbb{Z}_7 \times \mathbb{Z}_9$ isomorphic to $\mathbb{Z}_{21} \times \mathbb{Z}_3$? Explain.
   (b) Is $\mathbb{Z}_6 \times \mathbb{Z}_5$ isomorphic to $\mathbb{Z}_3 \times \mathbb{Z}_{10}$? Explain.

3. Prove that if G is a finite group of order n, and $g \in G$, then $g^n = e$.

4. Prove or disprove: *if $\phi : G \to K$ is a homomorphism, and $\psi : K \to L$ is a homomorphism, then $\psi \circ \phi$ is a homomorphism.*

5. Let $\phi : \mathbb{Z} \to G$ be a homomorphism.
   (a) Prove that if the order of $\phi(1)$ is infinite, then $\ker(\phi) = \{0\}$.
   (b) Prove that if the order of $\phi(1)$ is finite, then $\ker(\phi)$ is the cyclic subgroup of $\mathbb{Z}$ generated by the order of $\phi(1)$.

6. (*Hint for this problem*: the previous problem.)
   (a) Let ϕ be the (unique) homomorphism $\mathbb{Z} \to \mathbb{Q}$ for which $\phi(1) = -8.6$. Compute the kernel of ϕ.
   (b) Let ϕ be the (unique) homomorphism $\mathbb{Z} \to S_6$ for which $\phi(-1) = (13)(56)$. Give a formula for ϕ, and compute the kernel of ϕ.

7. Section 11 # 14, 24, 32 (not c or d), 36, 52