Homework 11

Math 416 (sections 200 [Honors] and 500 [Regular]), Spring 2017

This homework is due on FRIDAY, April 7 at noon[1]. *Write your section number on the top of your homework.*

0. *(This problem is not to be turned in.)*
   (a) Read Section 51
   (b) (Practice Problems) Section 51 #7, 11
1. True/False. (You do *not* need to give proofs for this problem.)
   (a) A finite extension K/F is a simple extension if and only if $G(K/F)$ is a simple group.
   (b) Every finite extension of a perfect field is simple.
   (c) $\mathbb{Q}(i, \sqrt[3]{2})$ is a splitting field of $\mathbb{Q}(\sqrt[3]{2})$.
   (d) If K is a splitting field of F, then *every* $f \in F[x]$ splits completely into linear factors in $K[x]$.
   (e) For any field F, every reducible polynomial $f \in F[x]$ has a root in F.
2. Section 51 #8, 14
3. Let E be the splitting field of $f(x) = x^3 - 3x + 1$ over $\mathbb{Q}$ (which you analyzed on HW 10, #7). Is there an automorphism of E for which exactly one root of $f(x) = x^3 - 3x + 1$ is fixed? Explain.
4. Is there an automorphism of the splitting field of $g(x) = x^3 - 4$ over $\mathbb{Q}$ for which exactly one root of g is fixed? Explain.
5. Consider field extensions $F \subset E \subset \overline{F}$. Let $\alpha \in \overline{F}$. *Prove or disprove*: if α is separable over F, then α is separable over E.
6. *Prove or disprove*: for a field F, the following is a subfield of $\overline{F}$:
$$\{\alpha \in \overline{F} \mid \alpha \text{ is separable over } F\} .$$
7. (Honors only!) Find $\alpha \in \overline{\mathbb{Q}}$ such that $\mathbb{Q}(\alpha) = \mathbb{Q}(\sqrt[4]{2}, i)$. Prove your answer.
8. (Honors only!) Prove your answers to the True/False above.

[1]You may turn in your homework to Blocker 601 E