Homework 15

Math 653, Fall 2019

This homework is due on TUESDAY, December 3.

1. Read Hungerford, Section 3.6
   (a) Section 3.5 #7
   (b) Section 3.6 #10
   (c) *(These problems are not to be turned in.)* Section 3.5 #2(a), 9
   (d) *(These problems are not to be turned in.)* Section 3.6 #5, 7; prove Theorem 6.1 on page 158
   (e) *(These problems are not to be turned in.)* Let R be a commutative ring. Let I be an ideal of R, and let (I) be the ideal of $R[x]$ generated by I. Prove or disprove: $R[x]/(I) \cong (R/I)[x]$. Also, if I is a prime ideal of R, does it follow that (I) is a prime ideal of $R[x]$?
2. Let R be a commutative ring, with prime ideal P. Let $S = R \setminus P$.
   (a) Prove that S is a multiplicative set.
   (b) Prove that $S^{-1}R$ has a unique maximal ideal. (Definition/Notation: $S^{-1}R$ is the *localization* of R at P, denoted by R_P. In general, a *local ring* is a commutative ring with a unique maximal ideal.)
3. Consider the polynomial $f = x^5 + 10x^4 + 25x - c$. For which $c \in \mathbb{Z}$ does Eisenstein's criterion imply that f is irreducible in $\mathbb{Q}[x]$? For those c, is f also irreducible in $\mathbb{Z}[x]$? Explain.