Homework 8

Math 653, Fall 2019

This homework is due on Thursday, October 17.

1. Read Hungerford, section 2.2.
   (a) Section 2.2 #2, 5, 12
   (b) *(These problems are not to be turned in.)* Section 2.2 #8, 11, 14
   (c) *(This problem is not to be turned in.)* What is the *torsion subgroup* of $\mathbb{C}^*$? Explain.
   (d) *(This problem is not to be turned in.)* Are $\mathbb{Z}_2 \times \mathbb{Z}_{20}$ and $\mathbb{Z}_4 \times \mathbb{Z}_{10}$ isomorphic?
   (e) *(This problem is not to be turned in.)* DETERMINE THE POSITIVE INTEGERS m AND n FOR WHICH THE FOLLOWING IS TRUE (AND PROVE YOUR ANSWER): For every abelian group G of order n, the following is a subgroup of G:
   $$H_m := \{0\} \cup \{g \in G \mid |g| = m\} .$$
   (How) does your answer change for non-abelian G?

2. (a) Give an example of an abelian group that is *not* finitely generated. (No proof necessary.)
   (b) List all subgroups of $\mathbb{Z}_{27} \times \mathbb{Z}_9$ of order 9. Prove your answer.

3. Assume m and n are positive integers with $\gcd(m, n) = 1$. Let N_m and N_n denote the number (up to isomorphism) of abelian groups of order m and n, respectively. How many abelian groups (up to isomorphism) of order $m \cdot n$ are there? Prove your answer.

4. *Complete the following claim, and give a proof:* A finite abelian group is _____ if and only if, for every prime p, the group does *not* contain a subgroup isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$.

5. (a) Which finitely generated abelian groups are *not* the internal direct sum of two or more nontrivial subgroups? Prove your answer.
   (b) (*Challenge – optional!*) What about abelian groups that are *not* finitely generated?

6. For each problem that you missed on the exam, re-do the problem. (If your answer on the exam was partially correct, you may write "On the exam, I proved ...; what remains to prove is ...".)

7. Write one paragraph reflecting on the exam. *How did you prepare for the exam? Did you study alone or with other students? What surprised you about the exam? How do you feel about your performance on the exam? Which types of errors (if any) did you make (not understanding the problem, not being able to formulate a solution, etc.)? Are there any concepts you want to work on before the next exam? Any other reflections or comments?*