Math 220 – Homework 1

Due Tuesday 01/30 at the beginning of class

Total points: 159 (Problems marked by $*$ will count toward writing portion.)

PART A

Problems from the textbook:

- Section 1.1

problem	2(e)	3(a,c)	7(a,b)$*$	9(d)	14(b,c)
points	5	10	10	5	10

PART B

1. [21 points] Determine whether each of the following sentences is a proposition, predicate, or neither.
   (a) $20^2 + 18^2 > 2018^2$
   (b) $x^2 = -1$.
   (c) For every real number x, $x^2 \neq -1$.
   (d) The product of every two prime numbers is odd.
   (e) Give an example of integrable function.
   (f) The plane is leaving in 20 minutes.
   (g) Excessive exposure to the sun may cause skin cancer.

2. [20 points] State the negation for each of the following propositions.
   (a) $\sqrt{3}$ is a rational number.
   (b) 0 is not a negative number.
   (c) The real number r is at most $\sqrt{3}$
   (d) Two sides of a triangle have the same length.
   (e) The point P on the plane lies outside of the circle C.

3. [32 points] Consider the following propositions

$$P : 2018 \in 3\mathbb{Z} \quad \text{and} \quad Q : 3^{2018} \in \mathbb{O}.$$

   Write each of the following compound statements in words and indicate whether it is true or false.
   (a) P; (b) Q; (c) $\neg P$; (d) $P \vee Q$; (e) $P \wedge Q$; (f) $P \Rightarrow Q$; (g) $\neg Q \Rightarrow P$; (e) $P \Leftrightarrow Q$.

4. $*$ [10 points] For the predicate $P(x) : (x^2 - 16)(x^4 - 16) = 0$, where $x \in \mathcal{U}$, determine:
   (a) the values of x for which $P(x)$ is a true statement if $\mathcal{U} = \mathbb{R}$ (Give reasons for your answer.)
   (b) the values of x for which $P(x)$ is a false statement if $\mathcal{U} = \mathbb{N}$.(Give reasons for your answer.)

5. [8 points] In each of the following statements identify the hypothesis (assumption) and conclusion.
   (a) If a is irrational, then $2a$ is irrational.
   (b) a^3 is an even integer whenever a is an even integer.

6. [16 points] Without changing their meanings, convert each of the following sentences into a sentence having the form "If P, then Q."

   (a) A function is integrable provided the function is continuous.

   (b) A function is rational if it is a polynomial.

   (c) "You fail only if you stop writing." (Ray Bradbury)

   (d) "Whenever people agree with me I feel I must be wrong." (Oscar Wilde)

7. [4 points] Without changing its meaning, convert the sentence*If a function has a constant derivative, then it is linear, and conversely.* into a sentence having the form "P if and only if Q."

8. [8 points] Prove that the statement $\neg((\neg Q \wedge (P \Rightarrow Q)) \Rightarrow (\neg P))$ is a tautology, a contradiction, or neither. You must state which of the three it is as well as give the proof.

9. $*$ [5 points] The professor tells to Amy: "If you get at least B on the final exam, then you will pass the course". Amy passes the course. What can she conclude?

   (a) She got at least B on the final exam.

   (b) She cannot conclude anything.

   Give reasons for your answer.

10. $*$ [5 points] The professor tells to Amy: "If you get at least B on the final exam, then you will pass the course". Amy finds out that she got a C on the final. What can she conclude?

    (a) She'd better start looking for a summer school course.

    (b) There's still hope.

    Give reasons for your answer.